

Sequential Spatiotemporal Magnetic-Field Reconstruction via Quantum Hamiltonian Learning with NV-Center Spin-1 Hamiltonians

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Abstract. We propose a sequential reconstruction framework for dynamic two-dimensional magnetic-field maps in which local observations are generated through nitrogen-vacancy (NV)-center spin-1 dynamics rather than direct field readout. A rotating-frame full spin-1 Hamiltonian defines the likelihood for each local window, while sequential Monte Carlo (SMC) jointly estimates local field values and a shared dipolar coupling parameter. Adaptive controls are selected by expected information gain followed by Fisher-information and leakage-aware scoring. Numerical proof-of-concept experiments on a structured 60×60 field over 16 frames reduce the root-mean-square error (RMSE) from 1.396×10^{-6} T to 7.037×10^{-7} T, with final Dice and intersection-over-union (IoU) scores of 0.9930 and 0.9860. Coupling-sensitive controls increase the mean coupling classical Fisher information by $22.5\times$ but increase leakage by $50.0\times$. The final coupling posterior is narrow ($J_{\text{std}} = 87.0$ Hz) but biased by 326.9 Hz, identifying joint field-coupling identifiability as the main limitation.

1. Motivation and contribution

Quantum Hamiltonian Learning (QHL) estimates latent physical parameters from measurement statistics generated by candidate Hamiltonians [1, 2]. We extend this principle from low-dimensional Hamiltonian identification to a distributed inverse problem: reconstruction of a magnetic field $B_t \in \mathbb{R}^{H \times W}$ that evolves over discrete frames t . The field is observed only through overlapping local windows, and each observation is a nonlinear function of local field values and a shared interaction parameter. This setting connects QHL, adaptive Bayesian inference, particle filtering, and NV-based quantum sensing [3, 4, 5].

The principal contributions are: (i) a local likelihood model derived from a full spin-1 NV Hamiltonian; (ii) joint particle-based inference of local magnetic fields and a global dipolar coupling J ; (iii) adaptive two-phase control selection that explicitly balances information gain, magnetic-field sensitivity, coupling sensitivity, and leakage; and (iv) sequential aggregation and temporal posterior propagation for dynamic field reconstruction. The study is a trusted-simulator proof of concept based on exact diagonalization, not a hardware demonstration.

2. Hamiltonian likelihood and sequential inference

For scan window s at frame t , let $\mathbf{b}_{t,s} = (b_1, \dots, b_M)$ denote the local field vector. The rotating-frame full spin-1 model is

$$H_{\text{NV}}(\mathbf{b}, J; \Omega) = \sum_{q=1}^M h_q(b_q; \Omega) + J \sum_{q < r} \frac{S_{x,q} S_{x,r} + S_{y,q} S_{y,r} - 2S_{z,q} S_{z,r}}{|q - r|^3}, \quad (1)$$

where h_q contains the field-dependent detuning relative to B_{ref} , site-dependent transverse-strain disorder, and microwave control. The numerical backend uses the positive sensitivity scale $\gamma_{\text{NV}} = |\gamma_e| = 28.025 \times 10^9$ Hz/T and retains the full $\{|+1\rangle, |0\rangle, |-1\rangle\}$ manifold so that leakage can be evaluated explicitly.

For a control $u = (T, \Omega, O)$, exact diagonalization gives $U(\mathbf{b}, J; u) = \exp[-iT H_{\text{NV}}]$. The effective binary Ramsey-readout probability is

$$p(\mathbf{b}, J; u) = \frac{1 + \langle \psi_0 | U^\dagger O U | \psi_0 \rangle}{2}, \quad (2)$$

and repeated measurements follow a binomial likelihood. Local field particles $\{\mathbf{b}^{(i)}\}$ and global coupling particles $\{J^{(j)}\}$ define joint candidate pairs whose weights are updated by Bayes' rule. Marginalization yields the local posterior mean and the updated global posterior over J .

Each window uses 12 adaptive updates: six field-sensitive steps followed by six coupling-sensitive steps. Candidate controls are first ranked by expected information gain (EIG). The highest-ranked controls are then evaluated by a phase-dependent score containing classical Fisher-information terms and a penalty for leakage from the effective sensing subspace. The field-sensitive phase favors short Ramsey interrogations, whereas the coupling-sensitive phase permits longer times and a Bell-pair initialization backend to enhance interaction sensitivity.

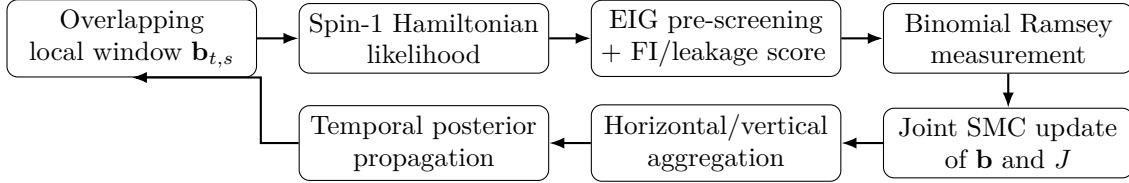


Figure 1: Sequential reconstruction loop. Local Hamiltonian-induced likelihoods support adaptive control, joint field-coupling inference, global aggregation, and frame-to-frame propagation.

Table 1: Representative configuration and final-checkpoint results.

Quantity	Value
Field sequence	60×60 , 16 frames, 50–55 μT
Local model	Full spin-1, $M = 6$, dipolar $1/ q - r ^3$ tail
Inference	256 local particles; 256 global J particles
Adaptive schedule	6 field-sensitive + 6 coupling-sensitive steps
Final RMSE / Dice / IoU	$7.037 \times 10^{-7} \text{ T}$ / 0.9930 / 0.9860
Final $\hat{J}$ / J_{std}	5326.9 Hz / 87.0 Hz
Absolute coupling bias	326.9 Hz
Product-state reference benchmark	73.3 Hz
Gain-extrapolated benchmark ratio	$J_{\text{std}}/26.0 \text{ Hz} = 3.35$

3. Numerical experiment

The benchmark uses a 60×60 maze-like field with 16 frames ($t = 0, \dots, 15$). The field spans 50–55 μT and evolves through frame-to-frame Gaussian perturbations with standard deviation 50 nT. Local windows have size $M = 6$; horizontal and vertical scans use stride two. The inference model maintains 256 local field particles and 256 global J particles, with $J_{\text{true}} = 5.0 \text{ kHz}$ and search range $[0, 10] \text{ kHz}$.

The field-sensitive controls use $T \in \{4, 8, 11\} \mu\text{s}$, whereas the coupling-sensitive phase additionally uses $T \in \{100, 157\} \mu\text{s}$. The latter controls amplify interaction-induced phase accumulation but also increase population transfer outside the effective sensing subspace. Reconstruction accuracy is evaluated by RMSE and mean absolute error for continuous amplitudes, and by Dice and IoU after thresholding the structured support.

4. Results

Figure 2 summarizes the reported frame-wise behavior. RMSE decreases monotonically from $1.396 \times 10^{-6} \text{ T}$ at $t = 0$ to $7.037 \times 10^{-7} \text{ T}$ at $t = 15$. Dice increases from 0.9279 to 0.9930, and IoU increases from 0.8654 to 0.9860. The results show progressive recovery of the dominant maze-like support and continuous field magnitude. Combined horizontal and vertical scans outperform either single-direction acquisition in the tested setting, indicating that complementary scan geometry reduces directional artifacts.

EIG decreases across the 12 adaptive steps, with most posterior reduction occurring in the early measurements. This is consistent with local posterior concentration and diminishing returns from later measurements. The coupling-sensitive phase raises the mean summed magnetic-field classical Fisher information from 5.14×10^{13} to 2.28×10^{15} (44.4 $\times$) and the mean coupling classical Fisher information from 2.20×10^{-7} to 4.97×10^{-6} (22.5 $\times$). Mean leakage simultaneously rises from 5.16×10^{-8} to 2.58×10^{-6} (50.0 $\times$), demonstrating a sensitivity-leakage tradeoff in the full spin-1 model.

The shared coupling is substantially less identifiable than the field. At the final checkpoint, $\hat{J} = 5326.9 \text{ Hz}$ and $J_{\text{std}} = 87.0 \text{ Hz}$ for $J_{\text{true}} = 5000 \text{ Hz}$. The posterior width is close to the finite-time product-state reference benchmark of 73.3 Hz, but the absolute bias is 326.9 Hz, approximately 3.76 posterior standard deviations. The uncertainty is also 3.35 $\times$ above the gain-extrapolated ideal-state benchmark of 26.0 Hz. Thus, posterior concentration alone does not establish unbiased parameter recovery.

5. Discussion and outlook

The field-coupling asymmetry follows from the likelihood geometry. Local fields directly determine dominant phase accumulation, while J enters indirectly through weaker interaction-mediated evolution. Different field-coupling combinations can therefore produce similar measurement probabilities, allowing the particle posterior to concentrate around a biased coupling mode. The result is a useful negative finding: the reconstruction architecture is effective for the tested field class, but the current control and readout set is insufficient for reliable joint recovery of J .

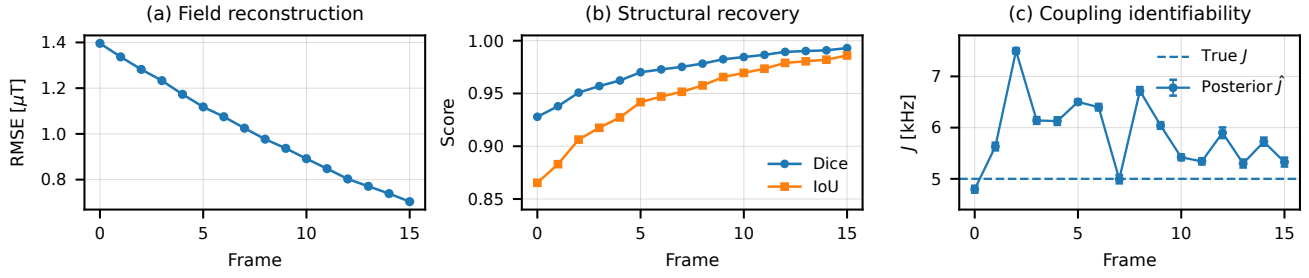


Figure 2: 16-frame numerical results. (a) RMSE decreases monotonically. (b) Dice and IoU approach unity. (c) The posterior mean of J is frame dependent and biased relative to $J_{\text{true}} = 5.0$ kHz; error bars show the reported posterior standard deviation.

The metrological comparison is a finite-resource diagnostic, not a scaling claim. The experiment uses a fixed window size $M = 6$, product-state Ramsey initialization in the field-sensitive phase, a Bell-pair backend in the coupling-sensitive phase, and local or two-site readout. The reported optimal-to-product QFI gain is therefore a fixed-size benchmark and does not demonstrate Heisenberg-limit scaling [6]. A scaling study would require systematic variation of spin number, shot count, interrogation time, and explicit state-preparation and measurement resources.

Future work will focus on richer interaction-sensitive controls, explicit leakage-associated measurement outcomes, leakage-aware likelihoods, broader field morphologies, faster temporal dynamics, calibration drift, and model mismatch. Computationally, repeated full spin-1 exact diagonalization motivates surrogate likelihoods and hybrid effective/full-model inference. Experimental deployment will additionally require hardware-specific readout, decoherence, and control-error models.

6. Conclusion

Hamiltonian-induced likelihoods can serve as local observation models in a sequential machine-learning pipeline for distributed, time-dependent magnetic-field reconstruction. The proposed combination of full spin-1 NV dynamics, adaptive Bayesian design, joint SMC inference, spatial aggregation, and temporal propagation achieves stable reconstruction of the tested structured field sequence. The main unresolved issue is not spatial accuracy but joint field–coupling identifiability, together with the sensitivity–leakage tradeoff introduced by long coupling-sensitive interrogations.

Keywords: Quantum Hamiltonian Learning; NV centers; sequential Monte Carlo; Bayesian inference; quantum sensing; spatiotemporal reconstruction.

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