

Native Measurement Based Variational Ansatz: Design and Analysis

1 Motivation And Challenges

Although the Variational Quantum Eigensolver (VQE) remains the leading framework for near-term quantum simulation, its implementation is conventionally bound to the circuit model [16, 24, 5]. This reliance introduces physical bottlenecks that are due to the computational model—e.g. physical layout constraints require swap-gate routing overheads—that are less acute or replaced altogether when using other computational models.

In this respect, Measurement-Based Quantum Computation (MBQC) offers a distinct alternative [18, 7]. Here, the required entanglement is embedded entirely within a static resource state (a graph state), while the logical computation is driven by single-qubit measurements and classical feedforward. This paradigm is natively suited to architectures like silicon photonics and aligns directly with measurement-based topological fault-tolerance schemes. However, utilizing MBQC for VQE requires a systematic method to construct and optimize parameterized states natively within this measurement-driven framework [8, 20, 13, 17, 12].

Formulation of VQE and MBQC. We consider a target Hamiltonian $H = \sum_i a_i Q_i$ acting on an n -qubit register, with $a_i \in \mathbb{R}$ and Pauli strings Q_i . VQE seeks to minimize the expectation value [16, 24, 5]:

$$f(\boldsymbol{\theta}) = \langle \psi(\boldsymbol{\theta}) | H | \psi(\boldsymbol{\theta}) \rangle \quad (1)$$

In MBQC, an open graph $\Gamma = (G, I, O)$ defines the computation, where $G = (V, E)$ is a finite graph, and $I, O \subseteq V$ represent the input and output vertex sets [7]. Given an input state $|\psi_{\text{in}}\rangle$ for qubits labelled by I , the open-graph state is prepared by initializing $V \setminus I$ in $|+\rangle^{\otimes |V \setminus I|}$ and applying controlled- Z (CZ) operations across all edges E :

$$|G(\psi_{\text{in}})\rangle = \left(\prod_{(u,v) \in E} CZ_{(u,v)} \right) \left(|\psi_{\text{in}}\rangle_I \otimes \bigotimes_{w \in V \setminus I} |+\rangle_w \right) \quad (2)$$

This state is uniquely characterized by $|\psi_{\text{in}}\rangle$ and the set of stabilizers $S_v = X_v \prod_{(w,v) \in E} Z_w$ for all $v \in V \setminus I$. Computations are then performed by measuring each non-output qubit $v \in V \setminus O$ in the basis $|\pm_{\theta_v}\rangle = (|0\rangle \pm e^{i\theta_v} |1\rangle) / \sqrt{2}$, where θ_v is chosen adaptively using previous measurement outcomes $m_u \in \{0, 1\}$. [7, 4].

MBQC-VQE Challenges. In general, the stochasticity of the single-qubit measurement outcomes m_v will not yield a deterministic transformation, even when measurement angles are chosen adaptively. This is only structurally guaranteed if the open graph admits certain topological conditions (*gflow*) [4]. However, satisfying these topological constraints while preserving variational flexibility is a non-trivial problem that has compromised previous MBQC-VQE proposals [8, 20]. Previous approaches have either bypassed the open-graph convention entirely, precluding standard determinism verification [8], or achieved determinism by directly translating rigid, hand-crafted circuit geometries, leaving minimal room to exploit alternative MBQC-native topologies [20, 12].

Furthermore, two operational bottlenecks emerge even in the presence of *gflow*:

- **Warm-Start Destruction:** Setting parameters to zero ($\boldsymbol{\theta} = \vec{0}$)—or in general to any other value—does not implement the identity; it evolves the input into a Clifford-scrambled state $C_\Gamma |\psi_{\text{in}}\rangle$, neutralizing any pre-computed low-energy initialization procedure attempting to speed-up convergence to the minimum energy state [19, 22, 6].
- **Opaque Expressivity:** No general analytical method exists to evaluate the trainability or gradient landscapes of these variational states prior to execution, preventing the use of adaptive ansatz schemes such as those utilized in ADAPT-VQE [11, 10].

This work resolves these difficulties by introducing a complete framework for open-graph ansatz design, an efficient randomized *gflow*-compatible open-graph generator, and classical structural diagnostics tools.

2 Technical Contributions

Composable Open-Graph Gadgets For Ansatz Preparation. We define a variational *open-graph gadget* as an open-graph $\Gamma = (G, I, O)$ with $G = (V, E)$ satisfying $|I| = |O|$ that admits a *gflow* $(g, \prec)$, and where each of the measurement angles $\theta_v, v \in V \setminus O$ can be varied in $[0, 2\pi]$.

The interest of gadgets is to attach them to a subsystem of n -qubit register $A \subseteq Q$ by defining a one-to-one mapping between A and I , which then yields a parametrized, deterministic transformation [21, 2] of I as part of Q into O . Identifying the isomorphic input and output Hilbert spaces via a canonical mapping, the state evolves as [21]:

$$|\psi_{\text{out}}(\boldsymbol{\theta})\rangle = \mathcal{U}_\Gamma(\boldsymbol{\theta})\mathcal{C}_\Gamma |\psi_{\text{in}}\rangle, \quad (3)$$

which resides on $(Q \setminus A) \cup O$. Here $\mathcal{C}_\Gamma = \mathbb{I}_{Q \setminus A} \otimes C_\Gamma$ is a parameter-independent Clifford unitary, and $\mathcal{U}_\Gamma(\boldsymbol{\theta})$ is a *Pauli Evolution Sequence* (PES) parameterized by the measurement angles $\boldsymbol{\theta} = (\theta_v)_{v \in V \setminus O}$ and ordered by the relation $\prec$:

$$\mathcal{U}_\Gamma(\boldsymbol{\theta}) = \prod_{v \in V \setminus O}^{\prec} \exp\left(-i\frac{\theta_v}{2}P_v\right). \quad (4)$$

The Pauli generators P_v , whose non-trivial action touches only qubits in O , and the Clifford frame C_Γ are determined only by the underlying topology of Γ and the gflow $(g, \prec)$ [21]. This Clifford+PES representation shows that gadgets can be concatenated or arranged in parallel, scaling the ansatz expressivity akin to increasing depth in circuit-based models.

Randomized open-graph generation. To bypass the restrictive, hand-crafted geometries of previous architectures, we introduce a stochastic open-graph generation algorithm. The framework relies on an efficient subroutine, **AddVertex**, which updates a graph $\Gamma = (G, I, O)$ to $\Gamma' = (G', I', O)$ by appending a single measured vertex v in a way that is guaranteed to extend the gflow of Γ . To this end, new edges are established via a neighborhood $e(v) \subseteq V$ such that $E' = E \cup \{(u, v) \mid u \in e(v)\}$. Identifying a valid pair $(e(v), g(v))$ of neighborhood and correction set that preserves the global gflow constraints requires solving an under-constrained linear system over $\mathbb{F}_2$ of size $O(|V|)$ [21, 15]. The structural randomness of the construction arises directly from sampling the solution space of this system¹.

Each invocation adds a new parameter to the PES: $\mathcal{U}_{\Gamma'}(\boldsymbol{\theta}') = \mathcal{U}_\Gamma(\boldsymbol{\theta}) \exp(-i\frac{\theta_v}{2}P_v)$ [21]. Moreover, given a specific P_v we can efficiently ascertain existence of (and when existing compute) a $(e(v), g(v))$ pair that realizes it, allowing effective Ansatz engineering via **AddVertex**. Much like exploring non-standard layer connectivity in randomized classical neural networks, this mechanism provides a systematic method to break the unhelpful spatial symmetries of rigid lattices, allowing us to dynamically scale ansatz expressivity without relying on a pre-existing circuit blueprint.

These random open-graphs can then be composed with existing gadgets. They can also be used as a way to generate a global ansatz that would act on all qubits of $|\psi_{\text{in}}\rangle$ at once; a situation reminiscent of the Randomly Wired Neural Network construction [23].

Initialization Strategies and Gradient Diagnostics. To recover the warm-start advantage lost to Clifford scrambling, we evaluate a modified, Clifford-rotated cost function $f_\Gamma(\boldsymbol{\theta})$ against a transformed Hamiltonian H_Γ :

$$f_\Gamma(\boldsymbol{\theta}) = \langle \psi_{\text{in}} | \mathcal{C}_\Gamma^\dagger \mathcal{U}_\Gamma(\boldsymbol{\theta})^\dagger H_\Gamma \mathcal{U}_\Gamma(\boldsymbol{\theta}) \mathcal{C}_\Gamma | \psi_{\text{in}} \rangle, \quad \text{where } H_\Gamma = \mathcal{C}_\Gamma H \mathcal{C}_\Gamma^\dagger \quad (5)$$

At $\boldsymbol{\theta} = \vec{0}$, we exactly recover the unperturbed reference energy $f_\Gamma(\vec{0}) = \langle \psi_{\text{in}} | H | \psi_{\text{in}} \rangle$. Because $\mathcal{C}_\Gamma$ is a Clifford operator, H_Γ is constructed efficiently via classical pre-processing of the constituent Pauli strings.

The gradient with respect to the k -th measurement angle at $\boldsymbol{\theta} = \vec{0}$ reduces to:

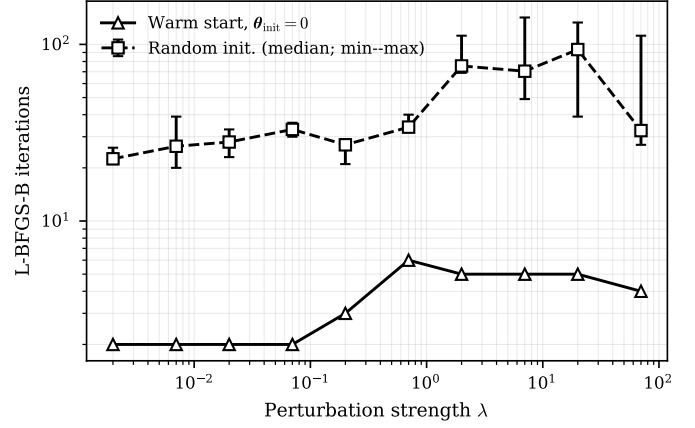
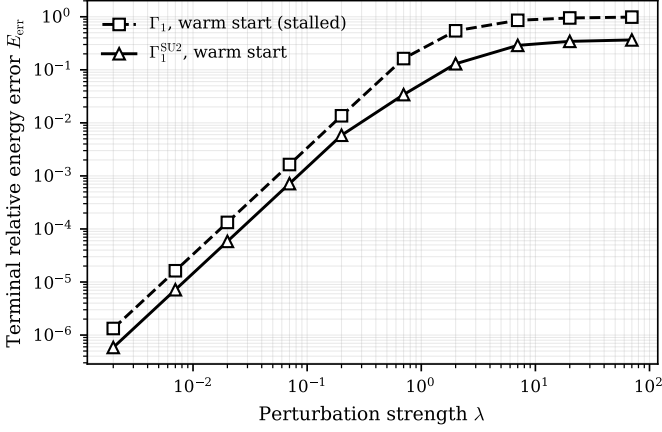
$$\left. \frac{\partial f_\Gamma}{\partial \theta_k} \right|_{\boldsymbol{\theta}=\vec{0}} = \frac{i}{2} \sum_j a_j \langle \psi'_{\text{in}} | [P_k, \tilde{Q}_j] | \psi'_{\text{in}} \rangle, \quad \text{where } |\psi'_{\text{in}}\rangle = \mathcal{C}_\Gamma |\psi_{\text{in}}\rangle \text{ and } H_\Gamma = \sum_j a_j \tilde{Q}_j \quad (6)$$

The commutator $[P_k, \tilde{Q}_j]$ is non-vanishing if and only if the generator P_k anti-commutes with the rotated term $\tilde{Q}_j$. If $|\psi_{\text{in}}\rangle$ is a stabilizer state, these expectation values can be evaluated classically in polynomial time [9, 1]. This algebraic feature provides an efficient classical filter to certify candidate graph geometries before executing them on quantum hardware.

3 Numerical Validation and Broader Impact

Numerical Validation. We validate these techniques using perturbed stabilizer Hamiltonians $H(\lambda) = H_0 + \lambda H_\delta$, where H_0 is a stabilizer Hamiltonian and H_δ is a non-commuting perturbation [8]. When $\lambda \ll 1$, the ground state

¹Which differs in mechanism and context from procedures in [3, 14], that aim to increase measured vertices in open-graphs while preserving gflow.



(A) Effect of $SU(2)$ augmentation at the warm-start point $\theta = \vec{0}$.

(B) Optimization cost of abandoning $\theta = \vec{0}$ initialization.

Figure 1: Numerical illustration of optimization behaviour with varying λ . (A) Both Γ_1 and Γ_1^{SU2} are initialized at the warm-start point $\theta = \vec{0}$. Optimization for Γ_1 stalls at the starting point, whereas with Γ_1^{SU2} lower terminal energies are found. (B) shows that initializing the optimization at $\theta = \vec{0}$ requires substantially fewer L-BFGS-B iterations than random initialization $\theta \in [0, 2\pi]$.

is close to a stabilizer state, allowing a natural warm-start; while $\lambda \approx 1$, the ground state is expected to be highly non-Clifford, creating a rigorous testbed for optimization capabilities of the prepared Ansatz.

We confirm that evaluating the zero-angle ansatz $|\psi(\vec{0})\rangle = C_\Gamma |\psi_{\text{in}}\rangle$ against H_Γ preserves the warm-start energy landscape whereas evaluation against H shifts it significantly from the low energy region, for any Γ . We demonstrate practical utility of the gradient diagnostics and the gadget composability; when the optimization for a chosen gadget Γ_1 stalls at $\theta = \vec{0}$ due to the predicted gradient silence ($\nabla f_{\Gamma_1}|_{\theta=\vec{0}} = \vec{0}$) in (6), we recover gradients by composing local gadgets implementing an $SU(2)$ -rotation on each of the output qubits of Γ_1 . Specifically, the modified gadget Γ_1^{SU2} introduces new Pauli generators in the PES that prevent gradient silence at the initialization point and the optimizer converges to lower terminal energies (see Fig. 1A). We also demonstrate the optimization overhead of not starting the optimization at $\theta = \vec{0}$ in Fig. 1B.

Crucially, a near-identical gadget Γ_2 with minor structural differences remains gradient silent under the same $SU(2)$ -augmentation. Underscoring a key lesson: an Ansatz’s utility depends globally on its complete Clifford+PES representation and cannot directly be inferred from graphical similarity. In MBQC, structural modifications propagate through the global entanglement structure; hence, local patches must be evaluated by how they alter the global gflow relation rather than their graphical similarity alone.

Broader Impact. Our framework establishes an operational method that directly resolves the central limitations of existing MBQC-VQE schemes:

- **Determinism and Interpretability:** Unlike approaches that do not rely on gflow, we guarantee outcome independence via composability or via automated gflow-compatible open-graph generation, while the Clifford+PES mapping renders the physical transformation transparent. We also explicate why approaches deviating significantly from ours (like [8]) are prone to losing determinism and interpretability, making them less convenient for practical application.
- **Architectural Flexibility:** In contrast to templates restricted to rigid, hand-crafted geometries or specific fixed layers like MuTA [20, 12], we shift the paradigm toward broad, fluidly scalable open-graph families.
- **Algorithmic Structure and Scaling:** Driven by the AddVertex routine, our randomized generator puts MBQC-native ansatz design on similar footing as the circuit model, enabling structural growth and parameter adaptation without manual architectural adjustments. A plausible future direction is to make the open-graph generation aware of the problem Hamiltonian, by coupling the generation with adaptive search procedures like reinforcement learning or genetic algorithms.
- **Physical Optimization Leeway:** While analytically closest to MuTA [12], which is restricted to causal flow, our reliance on its generalization gflow permits measurement corrections to be distributed over arbitrarily sized sets of yet unmeasured vertices instead of just one. Physically, this relaxation allows more freedom in the temporal sequencing of the measurements, allowing a larger number of measurements to be executed in parallel. Overall, this grants significantly more physical leeway for execution on available platforms such as silicon photonics.

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