

Hamiltonian Decoded Quantum Interferometry

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Introduction. Two of the most promising applications for quantum computers are Hamiltonian optimization and Gibbs state sampling. On the optimization side, there has been a long line of work on algorithms for quantum optimization problems—the task of finding (or preparing) the approximate ground state of a local *Hamiltonian*; these include adiabatic/annealing paradigms [1, 2], variational and hybrid approaches [3, 4], quantum phase estimation [5, 6], filtering-based approaches [7, 8] and dissipative methods [9]. On the sampling side, there is a long line of work on preparing (or sampling from) Gibbs states, a central primitive across physics and chemistry. In classical statistical mechanics, Gibbs ensembles provide the foundation for computing macroscopic observables from microscopic models, enabling the estimation of free energies, partition-function proxies, and thermal expectation values [10–18]. However, classical sampling methods such as *Markov Chain Monte Carlo* (MCMC) [19] often suffer from long mixing times or sign problems, especially in frustrated systems or quantum many-body contexts.

A recent work introduced *Decoded Quantum Interferometry* (DQI) [20], a promising new quantum algorithm which naturally interpolates between sampling and optimization; specifically, by reducing these problems to a *quantum* state preparation task, which then itself reduces to a *classical* syndrome decoding problem. This algorithmic paradigm builds on Regev’s quantum reduction [21, 22], which goes back to seminal work of Aharonov and Ta-Shma [23], and which also appeared in the more recent breakthrough of Yamakawa and Zhandry [24]. At a high level, DQI finds approximate solutions to certain classical objective functions f over finite fields, such as $\mathbb{F}_2^n$ (e.g., max-XORSAT), by first preparing a state corresponding to a superposition of a degree- ℓ polynomial $\mathcal{P}$ evaluated at f , which is of the form

$$|\mathcal{P}(f)\rangle = \sum_{\mathbf{x} \in \mathbb{F}_2^n} \mathcal{P}(f(\mathbf{x})) |\mathbf{x}\rangle. \quad (1)$$

Once measured in the computational basis, this yields a biased sample, where a string x is obtained with probability proportional to $\mathcal{P}(f(\mathbf{x}))^2$, thereby enhancing the likelihood of finding a string with a large objective value.

The paradigm of DQI offers a fresh decoding-based perspective for classical optimization but is purely limited to diagonal Hamiltonians, which encode classical cost functions. This naturally raises a critical question:

Guiding Question

Can the decoding-based framework of DQI be extended to the quantum domain of general, non-diagonal and non-commuting Pauli Hamiltonians?

Our contribution. In this work, we answer this question in the affirmative by introducing Hamiltonian Decoded Quantum Interferometry (HDQI). HDQI is a strict and powerful generalization of DQI, and uses coherent Bell measurements and the symplectic representation of the Pauli group to reduce Gibbs sampling and Hamiltonian optimization to classical decoding and the preparation of a certain state characterizing the anti-commutation graph of the Hamiltonian. Consider a Hamiltonian on n qubits,

$$H = \sum_{i=1}^m v_i P_i, \quad (2)$$

where $m = \text{poly}(n)$, each $P_i \in \{1, X, Y, Z\}^{\otimes n}$ is a multi-qubit Pauli operator, and each $v_i \in \{\pm 1\}$ is an overall phase.¹ Denote the eigenvalues and eigenvectors of H by

$$\{\lambda \in \mathbb{R}, |\lambda\rangle \in \mathbb{C}^{2^n}\}_\lambda, \quad \text{such that} \quad H = \sum_\lambda \lambda |\lambda\rangle\langle\lambda|. \quad (3)$$

Our goal in this paper is to prepare mixed² quantum states that take the form of polynomials of Hamiltonians, i.e.

$$\rho_{\mathcal{P}}(H) = \frac{\mathcal{P}^2(H)}{\text{Tr}[\mathcal{P}^2(H)]}, \quad (4)$$

where $\mathcal{P}$ is an arbitrary degree- ℓ polynomial. Choosing $\mathcal{P}(x)$ to approximate $\exp(-\beta x/2)$ yields Gibbs states at inverse temperature β ; other choices of $\mathcal{P}$ prepare approximate ground states, microcanonical ensembles, and other spectral filters.

HDQI prepares this state by following three steps:

1. **Preparing the pilot state:** Prepare a certain state which we term a “pilot” state, which carries the amplitudes of the eventual state we want to prepare. For commuting Hamiltonians this pilot state is simply a weighted superposition of Dicke states, as in [20]. In general, its amplitudes depend on the non-commutative expansion of $\mathcal{P}(H)$ into elementary symmetric polynomials, and is not necessarily always efficiently preparable. In the remainder of the algorithm, we will perform operations on the pilot state, tensored with the maximally-entangled state.
2. Controlled on the pilot state, we act with Paulis on half of the maximally-entangled state, creating a superposition over both registers with the desired amplitudes.
3. **Uncomputation via decoding:** To uncompute the pilot state (i.e. set it back to $|\bar{0}\rangle$ and discard it), we coherently run a decoder for a certain classical error-correcting code associated with H , which we call the *symplectic code* of H . If the target is to prepare $\rho_{\mathcal{P}}$ where $\mathcal{P}$ is a degree- ℓ polynomial, this decoder must be able to decode ℓ errors on the symplectic code of H .

Step 2 is always guaranteed to be efficient in our setting where $m = \text{poly}(n)$; the complexity of HDQI arises from Steps 1 and 3. In other words, HDQI is efficient for polynomial $\mathcal{P}$ whenever it is efficient to prepare the pilot state corresponding to $\mathcal{P}$, *and* there exists an efficient decoder in the sense stated in Step 3.

This process relies on two key technical ingredients. The first is the **Symplectic Code**, denoted $\text{Symp}(H)$, which is the classical linear code whose parity check matrix is formed by the $2n$ -bit symplectic representations of the Hamiltonian’s m Pauli terms. To construct this code, we rewrite each Pauli P_i in its *symplectic representation* $\text{Symp}(P_i) \in \mathbb{F}_2^{2n}$, which is a bitstring whose first n bits record the Z -support and the last n bits record the X -support of P_i . Modulo overall phases, this mapping is a group isomorphism:

$$\text{Symp}(P_i P_j) = \text{Symp}(P_i) \oplus \text{Symp}(P_j) \in \mathbb{F}_2^{2n}. \quad (5)$$

The symplectic code of H , denoted $\text{Symp}(H)$, is a classical linear code whose parity check matrix is given by binary matrix $B^T \in \mathbb{F}_2^{2n \times m}$ whose columns are $\mathbf{h}^{(i)} = \text{Symp}(P_i)$, i.e.

$$B^T = \begin{bmatrix} | & | & & | \\ \mathbf{h}^{(1)} & \mathbf{h}^{(2)} & \dots & \mathbf{h}^{(m)} \\ | & | & & | \end{bmatrix}. \quad (6)$$

If H is diagonal, the second n rows of B^T are zero and can be ignored: In this case, we recover the standard decoding problem of DQI. The second is the use of the **maximally entangled state** $|\Phi^n\rangle = \frac{1}{2^{n/2}} \sum_{\mathbf{x}} |\mathbf{x}\rangle \otimes |\mathbf{x}\rangle$. This choice elegantly solves two fundamental problems that arise when moving from diagonal to non-diagonal Hamiltonians. First, since $|\Phi^n\rangle = \frac{1}{2^{n/2}} \sum_\lambda |\lambda\rangle \otimes |\bar{\lambda}\rangle$ for any orthonormal basis $\{|\lambda\rangle\}$, it provides a uniform superposition over the (unknown) eigenbasis of H , bypassing the

¹While our paper originally restricted to signed Pauli Hamiltonians for simplicity, it was pointed out by [25] that our algorithm works as-is (when using the non-commuting general reduction) for real coefficients specified to some finite precision.

²Our quantum algorithm naturally prepares a minimal purification of these density matrices, providing additional applications.

need to prepare this difficult state directly. Second, applying a Pauli operator $(P \otimes I)$ to $|\Phi^n\rangle$ produces states that are orthogonal for different Paulis, i.e., $\langle \Phi^n | (Q^\dagger \otimes I) (P \otimes I) | \Phi^n \rangle = \delta_{P,Q}$. This orthogonality allows for perfect discrimination via a coherent Bell measurement, making the final decoding step information-theoretically possible.

Key results. Our applications are threefold. First, HDQI yields provably efficient Gibbs sampling at *any* temperature for a broad class of commuting Hamiltonians, including 2D stabilizer codes, the color code, cluster Hamiltonians, and 3D models such as Haah’s code. Second, we show that for the *same* class of Hamiltonians there is also an efficient *classical* sampling algorithm, sharpening the boundary between quantum and classical tractability. Third, we extend HDQI to non-commuting Hamiltonians, where efficiency depends on the structure of the anti-commutation graph, and apply this to spin glasses and stabilizer code Hamiltonians with quantum defects. In several cases—including Haah’s code and the 2D toric code—the ability of HDQI to prepare Gibbs states subsumes prior results such as Ding et al. (FOCS 2024).

Key Results I: Commuting Hamiltonians and the Boundary of Classical Simulation. In the commuting regime, the pilot state is always efficiently preparable, so decoding becomes the main bottleneck. Our central result identifies a broad class of **nearly independent Hamiltonians**—commuting Pauli Hamiltonians whose symplectic code has constant dimension. For this class, HDQI efficiently prepares Gibbs states at any temperature and arbitrary spectral filters. For Haah’s code, this is the first proven efficient Gibbs sampler at arbitrary temperature we could find in the literature.

The same structural property also enables a new *classical* algorithm: one can efficiently sample stabilizer eigenstates according to the Gibbs distribution whenever the symplectic code has only $O(1)$ constraints. This “dequantization” result complements the quantum algorithm and precisely delineates the quantum-classical boundary.

Key Results II: Tackling Non-Commutativity via Graph Structure. For non-commuting Hamiltonians, the challenge is preparing the pilot state. We show that its complexity is governed by the Hamiltonian’s **anticommutation graph**. If this graph decomposes into components of size $O(\log n)$, the pilot state admits a Matrix Product State (MPS) form with polynomial bond dimension and matrices that are efficiently computable via dynamic programming. These results imply that pilot states for anticommutation graphs with small connected components are efficiently preparable.

This framework applies to physically motivated models with defects:

1. **Semiclassical Spin Glasses:** Classical spin glasses where a constant fraction p of Z -terms are replaced by X -terms. HDQI efficiently samples Gibbs states up to a constant temperature if p is below a certain threshold constant.
2. **Defected Topological Codes:** Local stabilizer Hamiltonians perturbed by random non-commuting terms, where similar guarantees hold.

Outlook and Future Directions. A primary direction is to broaden the class of Hamiltonians for which HDQI is efficient. This involves identifying new families of non-commuting Hamiltonians that possess the requisite sparse anti-commutation graph structure needed for efficient pilot state preparation. A particularly compelling goal is to generalize the techniques used for arbitrary-temperature Gibbs sampling from the commuting to the non-commuting regime.

Furthermore, future work can explore the full potential of the states prepared by HDQI. The algorithm naturally produces purifications like thermofield double states, which are more powerful than their mixed-state counterparts. Leveraging these states for tasks beyond simple eigenbasis sampling—such as estimating non-Clifford observables or studying quantum thermodynamics—remains a rich area for investigation.

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