

Scalable certification of coherent crosstalk through channel entanglement detection

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Summary. As quantum processors scale, understanding the structure of the noise affecting their operations becomes increasingly important for both error mitigation and quantum error correction [1, 2]. Although single- and two-qubit gate fidelities have improved significantly, correlated errors across multiple qubits, and coherent crosstalk in particular, remain present even in state-of-the-art devices [3]. At the same time, many commonly used characterization and mitigation methods assume that the noise can be described effectively as stochastic [4]. This is either assumed directly, as in recent large-scale experiments [5], or enforced using techniques such as randomized compiling [6] and Pauli twirling [7], which aim to convert coherent errors into an effective stochastic noise model. While these techniques simplify the treatment of noise, they introduce additional circuit and sampling overhead, and residual coherent or non-Pauli features may still remain. It is therefore useful to have a direct way of testing whether the observed noise is compatible with these simplified models. However, standard characterization methods often require full process tomography [8–10], whose cost grows exponentially with the number of qubits, or rely on strong assumptions about the form of the noise [11]. Crosstalk-detection protocols [12, 13], on the other hand, identify generic correlations between qubits but do not certify their quantum nature. This motivates scalable methods that use only partial information about the implemented channel.

In this work, we develop a certification framework to detect coherent crosstalk directly at the level of quantum channels. Our key idea is that coherent crosstalk corresponds to entanglement in the Choi state of the noise channel across the input-output pairs associated with different qubits, so that detecting crosstalk becomes an entanglement-detection problem. In this picture, the natural reference model is a separable Choi state, which still allows the local noise processes to be classically correlated but excludes entanglement between different qubit pairs. Incompatibility with this model therefore, certifies that the noise cannot be reproduced by a classically correlated mixture of local channels. This connection allows us to combine techniques from entanglement detection [14–20] with tools from noise learning [21–23]: we estimate channel correlators from randomized product-state preparations and local Pauli measurements, inverting the ideal gate in classical post-processing, and test their compatibility with the separable model through a hierarchy of moment-based semidefinite-programming relaxations [24]. The same construction extends to a stricter test of whether the observed data are compatible with a Pauli-channel model, providing a way to detect non-Pauli features of the noise. In both cases, at fixed relaxation order, the method requires only a polynomial number of moment variables for bounded-locality data and avoids full process tomography. We test the scalability and effectiveness of the method numerically on entangling unitary channels and amplitude-damping noise, showing that it detects both inter-qubit quantum correlations and deviations from a stochastic Pauli description. In particular, the certification remains conclusive in the presence of finite statistical precision, and we expect that the tests will be run on systems of tens of qubits. As a direct application, the framework can be used to assess the performance of randomized compiling by applying the certification tests before and after compilation, quantifying how much coherent crosstalk is suppressed and how closely the effective noise approaches a stochastic Pauli model.

Preliminaries We consider the noisy implementation of a target unitary $\mathcal{U}$ on N qubits. The experimentally realized channel is denoted by $\mathcal{C}_{\mathcal{U}}$, and its error component is defined as $\mathcal{E}_{\mathcal{U}} = \mathcal{C}_{\mathcal{U}} \circ \mathcal{U}^\dagger$. All the structural information about the noise is encoded in the Choi state $\mathcal{J}(\mathcal{E}_{\mathcal{U}})$ [25], in which each physical qubit defines one input-output Choi pair. The absence of inter-qubit quantum correlations then corresponds to separability of the Choi state across these pairs. This condition can be written as

$$\mathcal{J}(\mathcal{E}_{\mathcal{U}}) = \sum_k p_k \bigotimes_{i=1}^N J_k^{(i)}, \quad (1)$$

where $p_k \geq 0$, $\sum_k p_k = 1$, and each $J_k^{(i)}$ is a valid two-qubit Choi state for the local pair i . The shared index k allows classical correlations between the local channels but forbids entanglement between different pairs. We refer to

Eq. (1) as the separable-channel model, and incompatibility with it is precisely our definition of coherent crosstalk. A stricter description of the noise is a Pauli channel, $\mathcal{E}_P(\rho) = \sum_{\vec{\alpha}} q_{\vec{\alpha}} P_{\vec{\alpha}} \rho P_{\vec{\alpha}}$, whose Choi state is always of the form of Eq. (1) with additional local structure. Their Choi states are mixtures of products of local Bell states, so Pauli channels form a structured subset of separable channels. Compatibility with a Pauli model is therefore a strictly stronger requirement, and ruling it out is the statement that no stochastic Pauli description of the observed noise exists. The input to our method is a collection of channel correlators

$$C_{\alpha} = \text{Tr}(\mathcal{J}(\mathcal{E}_{\mathcal{U}})O_{\alpha}), \quad (2)$$

where the O_{α} are Pauli observables acting on one or a few Choi pairs. These correlators constitute only partial information about the channel: we neither require nor attempt full process tomography.

Coherent crosstalk detection. We now describe how the compatibility questions above are turned into a tractable optimization problem. The protocol has three main steps: i) measure experimentally accessible Pauli correlators of the channel (or equivalently its Choi state), ii) impose physicality constraints on the channel parameters so that they represent a valid quantum process, and iii) formulate the measured correlators as candidate moments of a probability distribution over this region. The existence of a compatible noise model can then be tested through a semidefinite relaxation of the corresponding moment problem.

i) Experimental access to correlators. The correlators in Eq. (2) are local Pauli expectation values on the Choi state, and are therefore directly accessible with classical shadows combined with a suitable classical post-processing [23]. Importantly, no physical preparation of the Choi state is needed. By the Choi–Jamiołkowski isomorphism,

$$\text{Tr}(\mathcal{J}(\mathcal{E}_{\mathcal{U}})U|\vec{r}\rangle\langle\vec{r}|U^{\dagger} \otimes V|\vec{s}\rangle\langle\vec{s}|V^{\dagger}) = \text{Tr}(\mathcal{E}_{\mathcal{U}}(U|\vec{r}\rangle\langle\vec{r}|U^{\dagger}) \otimes V|\vec{s}\rangle\langle\vec{s}|V^{\dagger}) \quad (3)$$

where $|\vec{r}\rangle, |\vec{s}\rangle$ are computational-basis states and $U = \bigotimes_j U_j$, $V = \bigotimes_j V_j$ are tensor products of random single-qubit Clifford unitaries, so that $U|\vec{r}\rangle$ and $V|\vec{s}\rangle$ are products of random Pauli eigenstates. Expectation values on $\mathcal{J}(\mathcal{E})$ are obtained by preparing random Pauli eigenstates, evolving them through the channel, and measuring in random Pauli bases. Only single-qubit preparations and measurements are required, which makes the protocol feasible on current hardware. In practice, however, one samples from the noisy process $\mathcal{C}_{\mathcal{U}}$ rather than from the error channel itself $\mathcal{E}_{\mathcal{U}}$. We account for this entirely in classical post-processing, by inverting the ideal gate $\mathcal{U}$ at the level of the shadow reconstruction map. Denoting by $\mathcal{M}^{-1}$ the inverse of the measurement channel of the randomized scheme, which for random single-qubit Cliffords factorizes into single-qubit maps depolarizing maps [21], we define the rotated reconstruction map

$$\mathcal{M}_{\mathcal{U}}^{-1} := \mathcal{M}^{-1} \circ (\mathcal{U}^{\dagger} \otimes \mathbb{1}), \quad (4)$$

so that the correlators of $\mathcal{E}_{\mathcal{U}}$ are estimated from data generated by the device as it is. Importantly, $\mathcal{M}_{\mathcal{U}}^{-1}$ inherits the structure of $\mathcal{U}$, rather than factorizing into single-qubit maps. As a result, the efficiency of the protocol depends on the classical computability of $\mathcal{U}$ [26]. Since the O_{α} have bounded locality, the standard shadow guarantees apply: L correlators supported on at most w qubits can be estimated to accuracy ε with $\mathcal{O}(4^w \varepsilon^{-2} \log(L/\delta))$ samples. Each estimate C_{α} thus comes with a statistical uncertainty ϵ_{α} , which we carry explicitly into the compatibility tests below.

ii) Physicality constraints. To check compatibility of the measured correlators with the separable model in Eq. (1), we must specify which local parameters correspond to physically valid channels. Each local Choi state in Eq. (1) can be parametrized in Bloch-tensor form,

$$J_k^{(i)} = \frac{1}{4} \left(\mathbb{1} \otimes \mathbb{1} + \vec{a}_k^{(i)} \cdot \vec{\sigma} \otimes \mathbb{1} + \mathbb{1} \otimes \vec{b}_k^{(i)} \cdot \vec{\sigma} + \sum_{\mu, \nu} T_{k, \mu \nu}^{(i)} \sigma_{\mu} \otimes \sigma_{\nu} \right), \quad (5)$$

where $\vec{a}_k^{(i)}$ and $\vec{b}_k^{(i)}$ are the Bloch vectors of the output and input qubit, and $T_k^{(i)}$ is the correlation matrix. For $J_k^{(i)}$ to represent a physical local channel, it must be the Choi state of a CPTP map. Trace preservation fixes $\vec{b}_k^{(i)} = 0$, while complete positivity is equivalent to $J_k^{(i)} \succeq 0$. Since $J_k^{(i)}$ is a 4×4 Hermitian matrix, this positivity condition can be written in terms of the elementary symmetric polynomials of the spectrum $\{S_r\}_{r=1,2,3,4}$ [27]. Using Newton identities, these S_r can be expressed in terms of $\text{Tr}((J_k^{(i)})^r)$, and hence as polynomial constraints on $\vec{a}_k^{(i)}$, $\vec{b}_k^{(i)}$, and $T_k^{(i)}$. These constraints, together with $\vec{b} = 0$, define the local feasible set $K_{\text{sep}}^{(i)}$ used in the separable-channel test. For the Pauli test, unitality further imposes $\vec{a} = 0$, the correlation matrix must be diagonal, and the corresponding Pauli eigenvalues must lie inside the tetrahedron defined by the Fujiwara-Algoet conditions [28]. This defines the smaller set $K_P^{(i)} \subset K_{\text{sep}}^{(i)}$. Crucially, all of these constraints are polynomial in the local parameters, which is exactly the structure the relaxation below exploits.

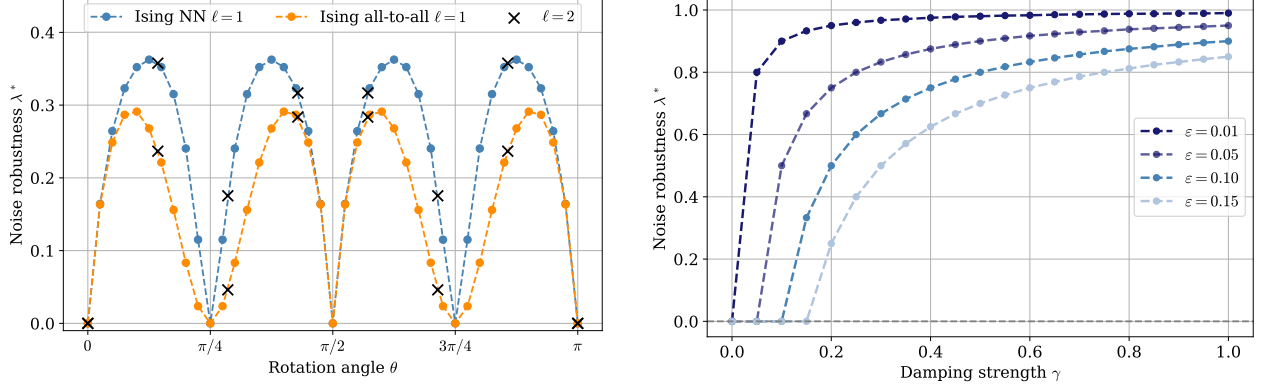


FIG. 1: (Left) Noise robustness λ^* of the separability test for the coherent error generated by an Ising-type unitary, as a function of the interaction angle θ . (Right) Robustness of the Pauli-channel test for the two-qubit amplitude-damping channel as a function of the damping strength γ , for different measurement imprecisions ε .

iii) **The compatibility test as an SDP relaxation.** If the data is compatible with the separable-channel model, the correlators must arise as moments of a probability distribution over the local feasible sets,

$$C_\alpha = \sum_k p_k f_\alpha(\xi_k^{(1)}, \dots, \xi_k^{(N)}), \quad (6)$$

where $\xi_k^{(i)} = (\bar{a}_k^{(i)}, \bar{b}_k^{(i)}, T_k^{(i)})$ and f_α is the polynomial in the Bloch-tensor variables associated with the expectation value of O_α . Deciding whether such a distribution exists is a constrained moment problem [20, 29]. We relax this problem to a semidefinite program by introducing a moment matrix Γ whose entries are moments of monomials m in the local variables up to degree ℓ , $\Gamma_{m_\alpha, m_\beta} = \langle m_\alpha m_\beta \rangle_P$. The entries of Γ corresponding to measured correlators are fixed by the experimental data, while the remaining moments are treated as optimization variables. A necessary condition for these moments to arise from a valid distribution is $\Gamma \succeq 0$. The equality constraints, such as $\bar{b}^{(i)} = 0$, become linear relations on Γ , and the polynomial inequalities defining the feasible region $K_{\text{sep}/P}^{(i)}$ are enforced through localizing matrices, whose entries are again linear in the moments. If the resulting SDP is infeasible, no distribution over the assumed local region can reproduce the measured correlators, certifying coherent crosstalk for $K_{\text{sep}}^{(i)}$ and excluding a stochastic Pauli description for $K_P^{(i)}$. Increasing ℓ yields a hierarchy of increasingly stringent tests [30], while at fixed ℓ the size of Γ grows only polynomially with N for bounded-locality data.

To make the protocol quantitative, we add depolarizing noise until the data become compatible with the model. This rescales the correlators as $C_\alpha \mapsto (1 - \lambda)C_\alpha$, and we solve

$$\begin{aligned} \lambda^* &= \min_{\lambda, \Gamma} \lambda \\ \text{s.t. } & \Gamma \succeq 0, \quad \Gamma \text{ satisfies the constraints of } K_{\text{sep}/P}^{(i)} \\ & (1 - \lambda)C_\alpha - \epsilon_\alpha \leq \tilde{C}_\alpha \leq (1 - \lambda)C_\alpha + \epsilon_\alpha \quad \forall \alpha, \end{aligned} \quad (7)$$

where $\tilde{C}_\alpha$ denotes the corresponding moment-matrix entry and the interval accounts for the statistical uncertainty ϵ_α of each correlator. A strictly positive λ^* certifies incompatibility with the model, and its magnitude quantifies how robust the certificate is to white noise and finite experimental precision.

Numerics We illustrate the protocol on two- and three-qubit systems. Although only these small instances are shown, our implementation of the separability test reaches 14 qubits. Since the computational bottleneck at this scale is the generation of the input data rather than solving the SDP, we expect the optimization to remain tractable for systems of several tens of qubits. Figure 1 (Left) shows the separability test for coherent Ising-type errors $e^{-i\theta H}$. The robustness λ^* is nonzero in the entangling regime, showing that partial correlator data can certify coherent crosstalk. Figure 1 (Right) shows the Pauli-channel test for the product amplitude-damping channel $\mathcal{A}_\gamma \otimes \mathcal{A}_\gamma$, which is local but is non-Pauli for $\gamma > 0$. The SDP detects this incompatibility even with finite statistical uncertainty ε in the correlators. In both cases, the first relaxation level augmented with a few higher-degree monomials is sufficient.

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