

Extended abstract of Thermal-Drift Sampling: Generating Random Thermal Ensembles for Quantum Chaos Diagnostics

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Introduction: The generation of random quantum states has emerged as a cornerstone of modern quantum science, playing a role analogous to random numbers in classical stochastic theories [1]. Ensembles of random pure states underpin our understanding of quantum complexity, chaos [2], and quantum metrology [3], with direct relevance to quantum circuits [4, 5], black-hole physics [6], and quantum supremacy experiments [7, 8].

Beyond pure states, thermal states (also known as Gibbs states) are central to statistical mechanics, which encodes equilibrium physics and underpins the studies of thermalization hypothesis [9], and spectral correlations predicted by the random matrix theory [9–11]. They exhibit complex correlations and volume-law entanglement, representing the typical mixed states found in quantum chaos and finite-temperature phases [4]. Preparing these states is computationally demanding: ground state preparation is known to be QMA-hard in the worst case [12, 13], with low-temperature thermal state preparation facing comparable complexity barriers [14]. While generating a specific thermal state can be intractable, this raises the question: can we efficiently prepare random structured thermal equilibrium states with their Hamiltonian labels?

Despite substantial theoretical progress, the practical generation of random thermal states remains unexplored. Current quantum methods typically rely on quantum phase estimation combined with imaginary-time evolution [15–19], fluctuation theorems [20], or quantum Metropolis and Lindbladian thermalization algorithms that couple the system to an effective bath [21–32]. These system-dependent methods are developed for preparing one thermal state, but do not directly address the task of sampling thermal states from an ensemble of Hamiltonians, as shown in Figure 1(a-b). In practice, extending these approaches to generate random thermal states requires repeatedly reconfiguring the algorithm for different Hamiltonian instances, with the randomness largely produced by classical preprocessing. Consequently, the overall pipeline for random Gibbs-state generation lacks global optimization and incurs substantial classical computational overhead. Our framework bypasses this by jointly generating the state and its label via mid-circuit measurement, though demonstrating a strict practical advantage over classical thermal-state samplers remains an interesting open question.

Overview of the results: In this article, we bridge this gap by introducing a dedicated quantum algorithmic framework for the scalable generation of random thermal states. In particular, we establish the following:

- (i) *Quantum thermal-drift channel:* We introduce the thermal-drift channel $\mathcal{N}_\sigma$ for any n -qubit Pauli operator σ , that applies a random non-unitary dynamic to the input state. This channel can be implemented with failure probability at most δ by repeating a map using $\mathcal{O}(n)$ quantum elementary gates and $2n$ ancilla qubits for $\log(1/\delta)$ times.
- (ii) *Measurement-controlled sampling algorithm:* We propose an efficient algorithm for sampling thermal states from an ensemble of Hamiltonians. The complexity of elementary gates scales polynomially with precision ε , inverse temperature β , and system size n .
- (iii) *Level statistics:* We benchmark our algorithm by applying level statistics analysis to the generated thermal state ensemble of an Ising model. The output distribution shifts toward Wigner–Dyson

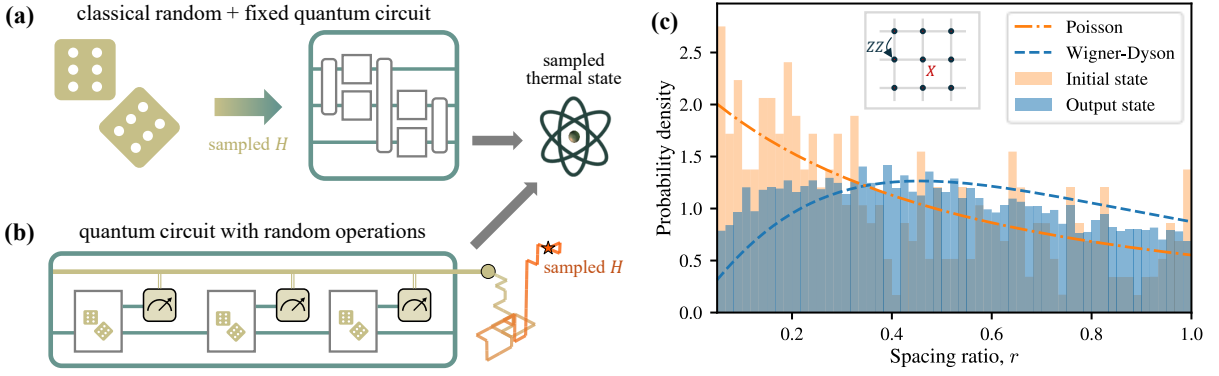


Fig 1: **(a-b)** Difference between classical sampling and measurement-controlled sampling. The task is to sample a thermal state together with its Hamiltonian label. (a) Classical workflow: sample label first, then prepare thermal state. (b) Our approach: sample state and label jointly via mid-circuit measurements. **(c)** Level-spacing ratio statistics for our sampling algorithm with respect to a 3×3 2D transverse-field Ising model. The output states (blue) shift toward the Wigner-Dyson prediction, indicating access to chaotic dynamics.

statistics, indicating that the sampler accesses a more chaotic sector of Hilbert space than the structured input with a many-body localization phase.

Quantum thermal-drift channel: One of our main contributions is the thermal-drift channel, the core component of our measurement-controlled sampling algorithm. This channel is implemented using system-ancilla coupling in a dilated Hilbert space, followed by measuring and tracing out the flag register, with only elementary gates. Given a Pauli operator σ and input state ρ , the channel is written in $\mathcal{N}_\sigma(\rho) = \frac{\mu}{2}(|\uparrow\rangle\langle\uparrow| \otimes \mathcal{E}_\sigma^\uparrow(\rho) + |\downarrow\rangle\langle\downarrow| \otimes \mathcal{E}_\sigma^\downarrow(\rho))$, where $\mu < 1$ is a scaling factor. Here, for any state ρ , the maps $\mathcal{E}_\sigma^\uparrow$ and $\mathcal{E}_\sigma^\downarrow$ are two completely positive and non-trace-preserving operations defined as $e^{+\tau\sigma/2}\rho e^{+\tau\sigma/2}$ and $e^{-\tau\sigma/2}\rho e^{-\tau\sigma/2}$, respectively, with a thermal strength $\tau > 0$. The implementation of $\mathcal{N}_\sigma$ is shown in Figure 2(a). The first register of $\mathcal{N}_\sigma(\rho)$ naturally generates a random branch. Measuring this flag register indicates which direction the state is thermalized toward, with probabilities proportional to the corresponding unnormalized traces. The resource complexity of this channel is summarized in the following theorem.

Theorem S7. For any n -qubit Pauli operator σ and $\tau > 0$, there exists a quantum channel $\mathcal{N}_\sigma$ implementable with $2n$ ancilla qubits and $\mathcal{O}(n \log(1/\delta))$ elementary gates with success probability $\geq 1 - \delta$. The circuit is shown in Figure 2(a).

Measurement-controlled sampling algorithm: Leveraging the thermal-drift channel, we implement our measurement-controlled sampling algorithm by applying N blocks to an initial state ρ_0 . In block k , a thermal-drift channel $\mathcal{N}_{\sigma_{j_k}}$ is applied, where σ_{j_k} is selected independently from the set Σ of L Pauli operators. The probability of choosing $\mathcal{N}_{\sigma_j}$ is weighted by the bound h_j , so that the distribution is h_j/λ . The thermal strength is fixed across all blocks to $\tau = \beta\lambda/N$. After the thermal-drift channel, a measure-and-trace operation $\mathcal{M}_{m_k}$ is applied to the flag register, generating a measurement outcome m_k . This outcome is recorded as $m_k \in \{+1, -1\}$ corresponding to $\uparrow$ and $\downarrow$. The overall algorithm is shown in Figure 2(b).

The full circuit is labeled by an ordered list of values $\mathbf{j} = \{j_1, \dots, j_N\}$ and an ordered list of directions $\mathbf{m} = \{m_1, \dots, m_N\}$, that corresponds to the channel $\mathcal{E}_{\mathbf{j}, \mathbf{m}} = \mathcal{M}_{m_N} \circ \mathcal{N}_{\sigma_{j_N}} \circ \dots \circ \mathcal{M}_{m_1} \circ \mathcal{N}_{\sigma_{j_1}}$. Accumulating the drift terms, the overall Hamiltonian label is given by $H_{\mathbf{j}, \mathbf{m}} = \frac{\lambda}{N} \sum_{j=1}^N \sum_{k:j_k=j} m_k \sigma_j$. The following theorem characterizes the sampling error and gate complexity.

Theorem 1. For any system size n , inverse temperature β , and sufficiently large N , our algorithm uses $\mathcal{O}(nN \log(1/\delta))$ gates to sample a thermal state and its label, with precision $\tilde{\mathcal{O}}(n^{3/2}\lambda^3\beta^3N^{-3/2})$ and failure probability at most δ .

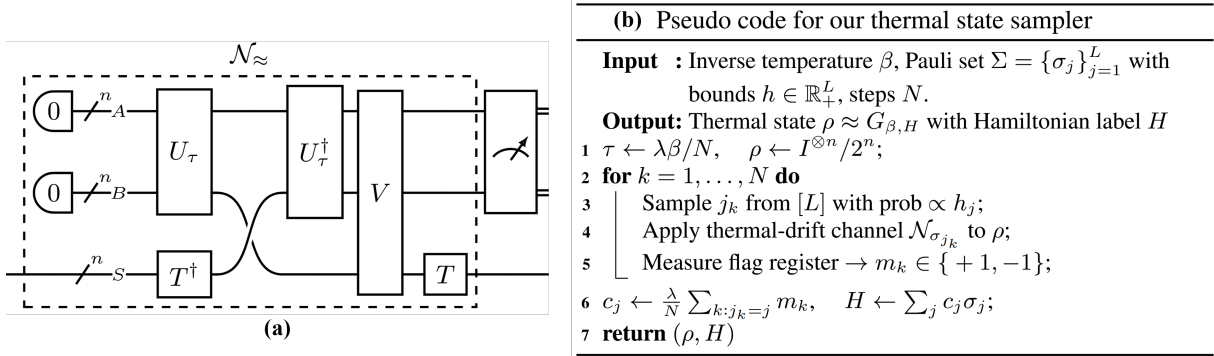


Fig 2: **(a)** Circuit implementing an approximate thermal-drift channel $\mathcal{N}_{\approx}$. Repeating until success (without postselection) simulates $\mathcal{N}_{\sigma}$. **(b)** Overview of the measurement-controlled sampling algorithm.

Since the sampling process is non-Markovian, the proof relies on martingale concentration [33] (Freedman inequality) rather than Trotter product formulas; when steps are sufficiently repeated, the measurement outcomes form a martingale sequence whose fluctuations can be bounded almost surely. Beyond precision guarantees, we also characterize the distribution of sampled labels.

Theorem 2. For any system size n and inverse temperature β , a label $H \in \mathfrak{H}$ can be asymptotically sampled with probability density proportional to an L -dimensional multivariate normal distribution reweighted by the partition function $\text{Tr}[e^{-\beta H}]$.

The proof models the sampling process as a random walk on an integer lattice, where each measurement outcome corresponds to a step; lattice-based random walk theory then yields the Gaussian approximation to the endpoint distribution, which implies a trade-off between sample accuracy and sample range. The algorithm does not naturally provide uniform coefficients or fixed disorder distributions, and avoids the worst-case exponential cost of traditional Gibbs sampling.

Level statistics: Level statistics provides a standard diagnostic for distinguishing chaotic (ergodic) quantum systems from integrable or localized ones [34]. In ergodic systems satisfying the eigenstate thermalization hypothesis, strong level repulsion leads to Wigner–Dyson statistics, whereas in the many-body localized phase, emergent local integrals of motion suppress correlations and produce Poisson statistics [10, 35]. The gap ratio $r_j = \min\{\delta_j, \delta_{j+1}\} / \max\{\delta_j, \delta_{j+1}\}$, where δ_j denotes adjacent level spacings, is insensitive to spectral unfolding and thus widely used in finite-size studies [11, 36–38].

We apply our algorithm to study the level statistics of the generated thermal state ensemble. We compare an initial reference state whose spectrum exhibits near-Poisson statistics with the output states produced by the algorithm for a transverse-field Ising model on a 3×3 2D grid. The result is shown in Figure 1(c). The output distribution shifts toward the Wigner–Dyson prediction, indicating that the sampler accesses a substantially more chaotic sector of Hilbert space than the structured input.

Conclusion: The ability to efficiently generate random thermal states for Hamiltonians drawn from broad Pauli ensembles opens several directions for future research in quantum many-body physics and quantum algorithms. On the physics side, our approach enables ensemble-level studies of thermalization, chaos, and many-body localization, providing direct access to level statistics, entanglement measures, and finite-temperature signatures without reliance on exact diagonalization or classical Monte Carlo methods [21]. This offers a scalable route to probing universality classes and crossover phenomena predicted by random matrix theory, opening new directions for ensemble-level thermodynamic studies.

From a computational perspective, random thermal states provide a physically grounded alternative to Haar randomness for benchmarking quantum devices, while remaining closely tied to experimentally relevant Hamiltonians. Our framework bridges quantum algorithms, statistical mechanics, and machine learning, providing tools for fundamental many-body physics and early fault-tolerant devices rather than near-term hardware. We expect these directions to further clarify the role of randomness, temperature, and complexity in interacting quantum systems.

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