

Training Variational Quantum Algorithms Is NP-hard, Even Locally

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Abstract. It is well known that globally optimizing the training objectives of Variational Quantum Algorithms (VQAs) is NP-hard. This motivates the use of local optimization methods in practice, raising the question of whether finding locally optimal parameters is computationally easier. We show that it is not: finding a local optimum for VQA training is also NP-hard. Our proof proceeds by establishing a bridge between polynomial optimization and VQA training, exploiting the connection between VQA objective functions and Hermitian trigonometric polynomials.

Keywords: Variational Quantum Algorithms · Complexity · Trigonometric Polynomials

Introduction

Variational Quantum Algorithms (VQAs) have emerged as an important framework for solving optimization and ground-state energy estimation problems on near-term quantum devices [1]. They have been widely studied for applications in quantum chemistry [2], finance [3], and cryptography [4]. VQA training consists of optimizing a parameterized quantum circuit (PQC) to minimize a loss function $f(\theta)$. This loss function represents the expectation value of a quantum observable with respect to the parameterized quantum state. Formally, the goal is to find parameters θ that solve $\min_{\theta} f(\theta)$. We refer to this problem as the VQA problem. Bittel and Kliesch have established that globally optimizing the training objective $f(\theta)$ of VQAs is NP-hard [5]. However, since VQA training typically relies on gradient-based methods, certifiable convergence to locally optimal solutions remains an active area of research.

In this work, we show that training VQAs is NP-hard, even locally. In particular, following ideas from Ahmadi et al. [6], who show that quadratic polynomial optimization is NP-hard even locally, we reduce the STABLESET problem to the problem of finding a local optimum of the VQA objective. Our proof technique exploits the relationship between the VQA objective function $f(\theta)$ and Hermitian trigonometric polynomials [7]. Table 1 summarizes existing results from the literature and highlights the gap addressed by our work. To the best of our knowledge, this is the first complexity result in the VQA training literature establishing the hardness of finding local optima.

Table 1: NP-hardness results for VQA and polynomial optimization across global and local optimization regimes. Prior to this work, the complexity of local optimization for VQAs (bottom-left) was open. Our result closes this gap.

	VQA	Polynomial
Global optimization	NP-hard (Bittel and Kliesch [5])	NP-hard (Pardalos and Vavasis [8])
Local optimization	NP-hard (Our result)	NP-hard (Ahmadi and Zhang [6])

Contributions

We consider a VQA objective for an N -qubit quantum system of the form

$$f(\theta) = \langle \psi(\theta) | H | \psi(\theta) \rangle, \quad (1)$$

where $H \in \mathbb{C}^{2^N \times 2^N}$ is a Hermitian matrix, and $|\psi(\theta)\rangle = U(\theta)|0\rangle$ is a parameterized N -qubit quantum state with trainable parameters $\theta \in [-\pi, \pi]^\ell$. Here, $U(\theta)$ is a parameterized quantum circuit formed by a sequence of fixed and parameterized unitary gates acting on the reference state $|0\rangle$. Our main result is the following theorem.

Theorem 1 *Finding local minima of the VQA problem $\min_\theta f(\theta)$ is NP-hard.*

Theorem 1 shows that the computational difficulty established by Bittel and Kliesch [5] persists even when the objective is relaxed from finding a global minimum to finding a local minimum. To prove this result, we exploit the fact that the objective $f(\theta)$ can be represented as a Hermitian trigonometric polynomial [5,9]. A Hermitian trigonometric polynomial is a function of the form

$$p(\theta) = \sum_{\alpha \in \mathbb{Z}^\ell} c_\alpha e^{i\alpha^\top \theta}, \quad (2)$$

where only finitely many coefficients c_α are nonzero, and $c_\alpha = \bar{c}_{-\alpha}$. Hermitian trigonometric polynomials arise naturally in harmonic analysis, with applications in Fourier analysis, signal processing, and approximation theory [10,11,7]. Through this connection, finding a local optimum of the VQA problem is equivalent to finding a local optimum of an appropriate Hermitian trigonometric polynomial $p(\theta)$.

To prove the complexity result, we show a polynomial-time reduction from the STABLESET problem, which asks whether a graph contains a collection of pairwise non-adjacent vertices of size k , to the VQA problem $\min_\theta f(\theta)$. The STABLESET problem is well known to be NP-complete [12]. Our reduction proceeds in two stages. We begin by reducing the STABLESET problem to finding a local optimum of a Hermitian trigonometric polynomial $p(\theta)$. In this part of the proof, we build upon the ideas of Ahmadi and Zhang [6] to show the following lemma of independent interest.

Lemma 2 *Finding local minima for a Hermitian trigonometric polynomial is NP-hard.*

Then we reduce the problem of finding a local optimum of $p(\theta)$ to finding a local optimum of the VQA objective $f(\theta)$ by showing that the coefficients of $p(\theta)$ can be realized as expectation values of unitary operators in polynomial time. Figure 1 summarizes the steps of the reduction.

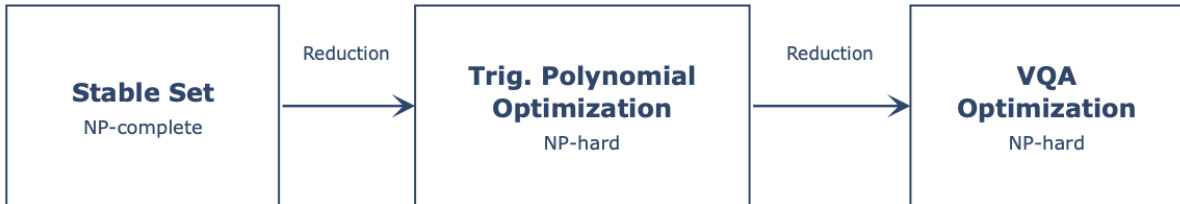


Fig. 1: Overview of the proof strategy for Theorem 1. We first establish the NP-hardness of local optimization for trigonometric polynomials via a polynomial-time reduction from the STABLESET problem. Since VQA objectives admit a trigonometric polynomial representation, this immediately implies the NP-hardness of local optimization for VQAs.

Why QTML?

Our talk speaks directly to a central challenge in the QTML community and provides further insight into the trainability of VQAs. We show that the difficulty of finding optimal parameters for VQAs cannot be bypassed even when the goal is to find only locally optimal solutions. We also demonstrate how techniques from well-established literature on polynomial optimization can be adapted to prove non-trivial results about the complexity of training VQAs. We believe this will be of independent interest to QTML attendees and will motivate further research in the field.

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