

Extended Abstract: Quantum Convolutional Neural Network for Phase Recognition in Two Dimensions

Leon C. Sander,¹ Nathan A. McMahon,^{1,2,3} Petr Zapletal,^{1,4} and Michael J. Hartmann^{1,5}

¹*Department of Physics, Friedrich-Alexander University Erlangen-Nürnberg (FAU), Erlangen, Germany*

²*(aQa^L) Applied Quantum Algorithms, Leiden University, Leiden, Netherlands*

³*LIACS, Leiden University, Leiden, Netherlands*

⁴*Department of Physics, University of Basel, Basel, Switzerland*

⁵*Quint Computing GmbH, Erlangen, Germany*

As quantum computers are becoming more powerful, the characterization of their output quantum states is a problem of increasing importance. Already existing noisy intermediate-scale quantum (NISQ) computers generate quantum states that are too complex to characterize with direct measurements and classical processing. While state-of-the-art measurement procedures, such as shadow tomography [1], efficiently characterize local properties, they require an unfeasable number of measurements for characterizing global features.

We present a solution to this challenge via direct processing of quantum data on the quantum computer in a Quantum Convolutional Neural Network (QCNN). Such networks have been shown to be a powerful tool in the context of condensed matter physics applications, especially for the classification of quantum phases of matter with reduced sample complexity. Yet, so far, QCNNs have only been considered for one-dimensional lattice systems, such as the cluster-Ising model [2], which can be efficiently described using classical numerical methods.

Our work introduces a QCNN for two dimensional systems, where classical numerics faces significant computational challenges. Here, classification is particularly challenging for topological many-body phases of matter due to the lack of local order parameters. We focus on detecting $\mathbb{Z}_2$ -topological order. Inspired by quantum error correction and multiscale entanglement renormalization ansatz (MERA), we analytically construct a QCNN and through matrix-product-state (MPS) simulations show, that it distinguishes the topological phase of the toric code in a magnetic field from a topologically trivial paramagnetic phase.

This design aims to reduce sample complexity, allowing for efficient state characterization on quantum hardware. Unlike snapshot-based data processing methods [3], our QCNN precisely identifies the critical value of the magnetic field for which the toric code undergoes the phase transition. Furthermore, it captures correlations between all stabilizer elements of the toric code, which cannot be accessed by direct measurements. This increases the threshold for errors leading to such correlations and allows for correctly identifying the topological phase in the presence of strong correlated errors. We finally show that the QCNN output is robust against incoherent errors below a threshold probability, which makes it feasible for implementation on current quantum computers under NISQ conditions.

QCNN DESIGN PRINCIPLES

Our QCNN consists of a convolutional layer composed of CNOT and Hadamard gates and pooling procedures implemented through controlled Pauli operations. Here, the principle for the phase detection relies on the definition of reference states for each phase. We use that local unitary perturbations on the reference states cannot drive them across a phase transition and therefore preserve the phase [5, 6]. Hence, we treat other input states from the same phase as the reference states masked by local unitary perturbations. We can then utilize concepts from quantum error correction to identify the phase of an input state by the removal of the local unitary perturbation and the subsequent recovery of a reference state.

During the convolutional layer, we map the toric code's stabilizer values to individual qubits, making single-qubit errors detectable. These errors induce pairs of syndromes in the toric code, which we correct through a specially designed pooling procedure consisting of l layers. In each layer, we progress to a smaller renormalized lattice by choosing a subset of qubits for further processing and discarding the others. If the

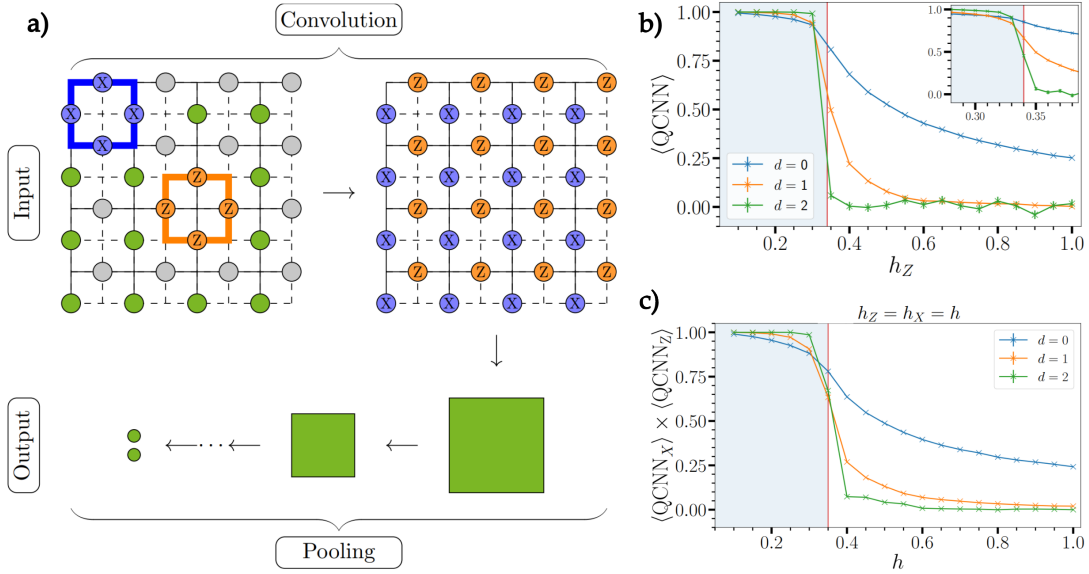


FIG. 1: a) In the first step the stabilizer values of the toric code are mapped to individual qubits. Subsequently, the lattice dimension is reduced with simultaneous error correction until only two qubits remain, which are measured to form the QCNN output. b) QCNN output for the toric code in a magnetic field in Z-direction. In the plot, the blue area marks the topological phase, which transitions to a Z-paramagnet for $h_Z^* = 0.34$. Here, we compare the measurement of the QCNN output for up to two layers of pooling $d = 2$ to a direct measurement $d = 0$. We find that with increasing depth its output converges to a step like function at the transition point. The inset shows the output with greater resolution at the transition. c) Detection of the multicritical point [4] in the presence of X- and Z-field with $h = h_X = h_Z$. We take the product of the pooling in the X- and Z-basis for each QCNN layer to show the combined phase recognition along the multicritical line $h_X = h_Z$.

input state is in the topological phase, this procedure ensures that each pooling layer reduces the density of errors on the lattice while it preserves the topological characteristic. This enables the QCNN to identify the topological phase in the final layer.

A key advantage of processing quantum data directly with the QCNN, rather than relying on direct measurements, is its ability to exploit correlations between stabilizer operators. With snapshot-based approaches such as locally error-corrected decoration (LED) [3], the plaquette operators A_p and star operators B_v cannot be measured simultaneously as they require readout in different bases. In contrast, our convolutional layer maps all stabilizers onto individual qubits that are read out simultaneously. This allows to account for correlations between syndromes analogously to correlated quantum error correction [7] in the pooling. This boosts the noise tolerance of the QCNN significantly, as we show in Figure 2 for a coherent field h_Y under incoherent Pauli-Y noise.

QCNN PERFORMANCE BENCHMARKING

To evaluate the performance of the QCNN, we perform several matrix product state (MPS) simulations of the toric code on an infinite cylinder with periodic boundaries in the vertical dimension l_2 . We first test the network on ground states of the Hamiltonian with a magnetic field in Z-direction. We observe that the QCNN output transitions at the critical field strength $h_Z^* = 0.34$, which we independently determine from the peak in the second derivative of the ground-state energy via infinite density matrix renormalization group (iDMRG). Furthermore, the output becomes increasingly step-like, signaling more precise phase recognition. We then investigate the behavior under a simultaneous magnetic field in X- and Z-direction and show the QCNN's ability to recognize the multicritical point [4]. We also explore the robustness of the QCNN output against incoherent Pauli noise, showing that it accurately recognizes the topological phase below a threshold error probability of $p_{\text{th}} = 2.28\%$ for Pauli-Z noise to $p_{\text{th}} = 3.48\%$ for Pauli-Y noise. As discussed above, the correlated pooling scheme further increases this threshold for correlated errors.

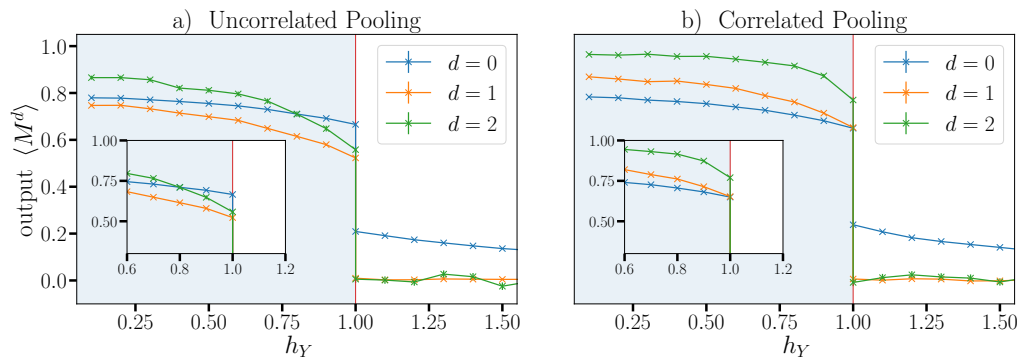


FIG. 2: Output of (a) uncorrelated pooling and (b) correlated pooling for the magnetic field strength h_Y in the presence of Pauli-Y noise with $p_Y = 3\%$ and for different depths d . Each data point represents 2000 samples from the MPS representation of the corresponding ground state with (a) the bond dimension $\chi = 1000$ and the periodic dimension $l_2 = 9$, and (b) the bond dimension $\chi = 1200$ and the periodic dimension $l_2 = 6$. This comparison shows that the correlated pooling scheme can precisely detect the first-order phase transition (vertical red line), whereas topological states close to the phase transition are misclassified for uncorrelated pooling, see the inset.

CONCLUSIONS

In summary, we explicitly constructed a QCNN that recognizes the topological phase of the toric code. Crucially, already with 2000 samples the sampling noise is negligible, see the error bars in Figure 1 and Figure 2. The QCNN output is robust to Pauli noise below a certain threshold probability, enabling the implementation on current quantum computers under NISQ conditions. This has been recently demonstrated on a superconducting quantum processor, using a convolutional layer equivalent to our construction, adapted to open boundary conditions and complemented by a classical postprocessing by a feedforward neural network [8, 9]. Additionally, the pooling layers can be effectively executed in classical post-processing, facilitating the QCNN implementation with limited circuit depths.

Future research could explore improvements to the quantum error correction procedures within the pooling layers, aiming to increase the error threshold and improve phase detection through quicker convergence to a step function. Implementing the convolutional layer as a quantum circuit paves the way for efficient quantum data processing with reduced sample complexity as it enables correlated pooling. We anticipate that our findings will inspire the use of QCNNs in characterizing less-understood quantum phases of matter, such as symmetry-enriched topological order [10] and quantum spin liquids [11].

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