

Parametrized-circuit-free quantum regression with variance regularization

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Introduction. Quantum regression—predicting scalar properties of quantum states from labeled training data—is commonly addressed using variational quantum algorithms. While variational quantum circuits are highly expressive and can generalize to unseen states with reasonable accuracy, they do not inherently encode the symmetries of the problem. We propose a complementary, problem-specific approach in which the relevant symmetries are built directly into the measurement ansatz, enabling more efficient and accurate solutions when such symmetries can be identified. Notably, this approach is entirely non-variational, requiring no parameterized quantum circuits.

Methodology and Results. Given a training set $\mathcal{T} = \{(\rho_i, \alpha_i)\}_{i=1}^T$ with real-valued labels, we seek a Hermitian observable H^* with $\alpha \approx \text{Tr}(H^*\rho)$ on unseen states. We expand H over M problem-inspired Hermitian operators, $H_\theta = \sum_j \theta_j G_j$, and fit them by jointly minimizing the least-squares error and the variance of H over the training set:

$$H^* \in \arg \min_H \left[w_{\text{ls}} \sum_i \left(\text{Tr } H \rho_i - \alpha_i \right)^2 + w_{\text{var}} \sum_i \left(\text{Tr } H^2 \rho_i - \text{Tr}^2 H \rho_i \right) \right]. \quad (1)$$

We validate our framework on two problems: 1. Estimating the transverse field strength in the Ising model, and measuring entanglement negativity in two-qubit bipartite systems. *For the Ising model*, we predict the relative field strength of an open 8-spin chain using a Pauli-string ansatz truncated by four constraints: nearest-neighbor locality ($l = 3$), open boundaries, spin-flip symmetry, and time-reversal symmetry (invariance under complex conjugation). This cuts the ansatz from over 1600 Pauli strings to $M = 90$, yet it still predicts accurately with low variance from as few as 10 training states. *For entanglement quantification*, we use four copies $\rho^{\otimes 4}$ with permutations acting independently on A and B . Four copies generate 100 Hermitian permutations, which can be grouped into 10 symmetry classes $\{\mathcal{C}_j\}$ —a compact 10-parameter ansatz observable $H = \sum_{j=1}^{10} \theta_j H_j$. Trained and evaluated on random mixed states, it outperforms variational approaches [1] in accuracy and efficiency (Fig. 1), and also generalizes from structured data—e.g. 50 pure plus 50 isotropic (*easy*) states—where variational methods and Pauli strings overfit or fail.

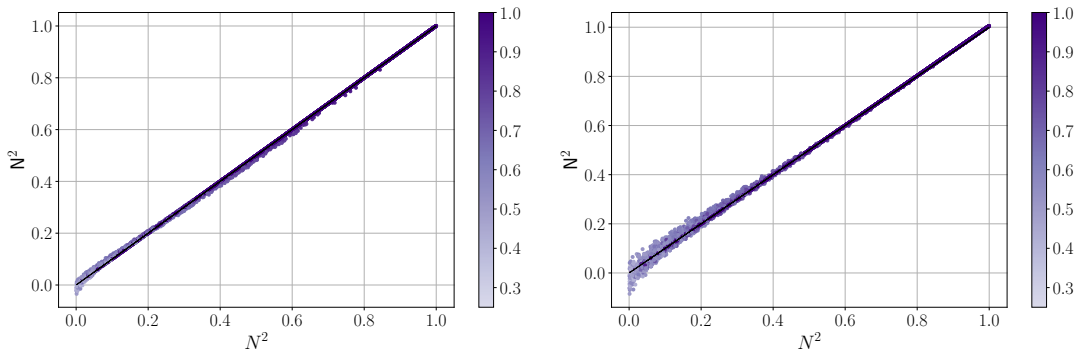


Figure 1: Predicted versus true squared negativity N^2 for random mixed states, using the 10-class Hermitian observable as the trainable model, trained on 100 *mixed* states (left) or 100 *easy* states (right).

References

- [1] A. Kardashin, Y. Balkybek, V. V. Palyulin, and K. Antipin, Phys. Rev. Research **7**, 013201 (2025).
- [2] Y. Balkybek, A. Kardashin, V. V. Palyulin, and K. Antipin, *Parametrized-circuit-free quantum regression with variance regularization*, arXiv:2607.02696 [quant-ph] (2026).