

A Mathematical Investigation of Quantum Kernel Methods for Non-Linear Classification

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Abstract

Kernel methods are fundamental to supervised machine learning because they enable the classification of non-linearly separable data through implicit mappings into higher-dimensional feature spaces. Recent advances in quantum machine learning have introduced quantum kernel methods, in which classical data are encoded into quantum states using parameterised quantum feature maps and similarity measures are computed through quantum state overlaps in Hilbert spaces. These methods have emerged as promising candidates for exploiting the representational capabilities of quantum systems while remaining compatible with established machine learning algorithms such as Support Vector Machines.

This work presents an ongoing mathematical and computational investigation of quantum kernel methods for non-linear classification. The study examines the theoretical foundations of classical and quantum kernel functions, focusing on feature-space geometry, Hilbert space representations, and the mathematical properties of quantum feature maps. A comparative computational framework is proposed in which classical kernel-based Support Vector Machines and quantum kernel classifiers are evaluated using benchmark non-linear datasets implemented on quantum simulators. The analysis will investigate classification behaviour, kernel matrix structure, and decision boundary characteristics to explore how quantum feature spaces differ from their classical counterparts.

Rather than assuming a quantum advantage, this research seeks to identify the mathematical conditions under which quantum kernel methods may provide distinctive feature-space representations and improved data separability. By integrating mathematical analysis with computational experimentation, the project aims to contribute to the theoretical understanding of quantum kernel methods and their application to supervised learning on near-term quantum computing platforms.

Keywords

Quantum Machine Learning; Quantum Kernel Methods; Support Vector Machines; Hilbert Spaces; Quantum Feature Maps; Machine Learning.

Selected References

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