

Multiscale Upsampling of Quantum States and Tensor Trains

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Extended Abstract

Quantum machine-learning and simulation algorithms often assume that data are encoded directly in quantum amplitudes or in q-sample states, in which probability distributions are represented coherently. Changing the resolution of such data, however, is nontrivial: explicitly refining a grid from 2^n to 2^{n+m} points incurs an exponential increase in representation size. We introduce a multiscale upsampling framework that refines quantum-state and tensor-train representations while keeping them compressed. Starting from an n -qubit state, or equivalently an n -core tensor train, the method produces an $(n + m)$ -qubit state or an $(n + m)$ -core tensor train defined on a finer grid.

Our main quantum contribution is a circuit construction for quasi-interpolating positive functions and probability distributions encoded in q-sample states. The refined state is prepared by a circuit of size $\text{poly}(n, m, p)$ using only $\lceil \log_2(p + 1) \rceil$ ancilla qubits, where p controls the smoothness and order of the interpolation kernel. For fixed kernel order, the ancilla requirement is constant, while the approximation error scales quadratically with the initial grid spacing, namely as $\mathcal{O}(h^2)$. The construction therefore enables quantum-encoded distributions to be refined coherently without explicitly materializing the exponentially larger fine grid.

The tensor-train formulation provides the classical tensor-network counterpart of the quantum procedure. It supports interpolation, quasi-interpolation, data augmentation, and multiscale field synthesis through low-rank contractions. The refinement cores introduced by the method have constant rank, the interpolation error is independent of the number of added grid points, and the computational cost grows only logarithmically with the final grid size at fixed accuracy. Consequently, the same framework can be used both as a compressed classical algorithm and as a blueprint for quantum-state preparation.

We validate the approach on representative one-, two-, and three-dimensional examples, including smooth functions and their derivatives, geometric airfoil masks, and random fields with turbulence-like structure. Across these settings, the method preserves multiscale features while avoiding explicit construction of the fine discretization. The results establish a direct connection between classical interpolation theory, tensor-network methods, and quantum-state representations. This connection enables resolution-aware quantum data loading, multiscale q-sample preparation, compressed scientific-field generation, and tensor-network-assisted quantum machine learning.

Keywords: quantum state preparation; q-sample states; tensor trains; matrix product states; multiscale interpolation; quasi-interpolation; quantum machine learning.