

Non-Markovian Noise Suppression Simplified through Channel Representation

Zhenhuan Liu, Yunlong Xiao, and Zhenyu Cai

The technical version can be found at arXiv:2412.11220.

Background. Noise presents the most significant obstacle to the practical implementation of quantum computation. A range of techniques has been proposed for noise suppression at different levels, including dynamical decoupling [1], quantum error correction (QEC) [2, 3] and mitigation (QEM) [4]. Most error suppression techniques are developed based on the assumption that the noise process is Markovian, i.e. the errors occurring at different time slices are independent and can be described as independent quantum channels. Under this assumption, one can develop QEC and QEM techniques to address the noise channels at a given time slice and simply repeat the QEC and QEM procedure. However, in practice, since noise is introduced by continuous interaction between the environment and the quantum computer, it is no surprise that the environment can exhibit memory effects that lead to quantumly correlated noise at different time slices [5–7]. This type of noise, known as non-Markovian noise, cannot be effectively suppressed by simply repeating the same QEC and QEM procedures.

Attempts have been made to study the role of non-Markovian noise in hardware noise suppression [1, 8–10], QEC [11–13] and QEM [14]. In many cases, one relies on constructing explicit models of the system-environment interaction or restricting to only classical temporal correlations, while in some other cases, the analysis is limited to only the abstract theoretical level without connecting to explicit practical protocols. The primary challenge in analysing general non-Markovian noise lies in its complex nature, which requires higher-order quantum operations for a complete description. These include frameworks such as quantum strategies [15], quantum combs [16, 17], process tensors [18, 19], or quantum circuit fragments [20, 21]. In contrast, most existing error suppression techniques are designed for noise modelled as quantum channels [22–24], making them unsuitable for direct application to non-Markovian scenarios [25–28].

In this work, we present a simple yet surprisingly general and effective solution: a mapping between non-Markovian quantum dynamics and quantum channels, which we call the Choi channel representation. This representation facilitates the direct application of error suppression techniques developed for Markovian noise to non-Markovian scenarios, as demonstrated through several examples. We begin by developing the χ -matrix representation and Pauli twirling protocol for non-Markovian noise, showing that Pauli twirling can eliminate the quantum temporal correlation in arbitrary non-Markovian quantum dynamics. Then, using Choi channels, we manage to construct explicit protocols for performing probabilistic error cancellation and purification-based QEM on general non-Markovian noise [29–33]. All of these examples have demonstrated the simplicity and effectiveness of using Choi channels to directly import Markovian noise suppression techniques for tackling non-Markovian noise, thereby opening up a broad spectrum of new possibilities for addressing non-Markovian noises.

Choi Channel. No quantum system operates in complete isolation. In a quantum computer, repeated interactions with the environment generate temporal quantum correlations, giving rise to non-Markovian noise that poses significant challenges to reliable information processing. The dynamics of such noise are inherently complex. In general, a single time slice of such noise cannot be adequately described by a quantum channel or even by more general linear maps [34–36]. We can model a noisy circuit using Fig. 1(a) where layers of gates, denoted as $\mathcal{U}_i$, are interlaced by layers of noisy interactions, denoted as $\mathcal{E}_i$. In general, these interactions $\mathcal{E}_i$ can be connected through the environment and thus can be quantumly correlated over time. We will denote the entire non-Markovian quantum dynamics as $\mathcal{E}$, which can be mapped to a quantum channel $\mathcal{C}_{\mathcal{E}}$ by shifting the input and output wires of the i -th time step to the i -th auxiliary register as shown in Fig. 1(b). We will call this the *Choi channel* of the non-Markovian quantum dynamics.

For simplicity, let us illustrate this using the example of a non-Markovian noise process $\mathcal{E}$ interacting at two different time steps, before and after the ideal gate layer $\mathcal{U}$, as shown in Fig. 1. Such non-Markovian noise affecting two points in time can be mathematically represented by a *superchannel* that maps one channel to another [37]. By applying the Choi channel of the noise $\mathcal{C}_{\mathcal{E}}$ to the Choi state of the ideal gate layer $\rho_{\mathcal{U}}$ and performing the relevant Bell state measurements, we can obtain the output channel representing the noisy gate layer. This approach closely mirrors the action of a conventional noise channel on some input state, which allows us to extend many

noise suppression techniques for Markovian noise to the context of non-Markovian noise, as we will demonstrate in subsequent sections. Our example here can be more formally stated as follows.

Definition 1. Given a superchannel $\mathcal{E}$, its Choi channel is defined as

$$\mathcal{C}_{\mathcal{E}} = \text{SWAP}^{(1,2)} \circ (\mathcal{E}^{(1)} \otimes \mathcal{I}^{(2)})[\text{SWAP}^{(1,2)}], \quad (1)$$

where $\mathcal{I}$ is the identity superchannel, 1 and 2 denote two quantum registers, and $\circ$ denotes the composition between channels. The resultant channel of acting $\mathcal{E}$ on the input channel $\mathcal{U}$ is

$$\mathcal{E}[\mathcal{U}](\rho) = \text{Tr}_{2,R} \left\{ (\mathbb{I}^{(1)} \otimes \Phi_+^{(2,R)}) [(\text{SWAP}^{(1,2)} \circ \mathcal{C}_{\mathcal{E}}^{(1,2)}) \otimes \mathcal{I}^{(R)}] (\rho^{(1)} \otimes \rho_{\mathcal{U}}^{(2,R)}) \right\}, \quad (2)$$

where $|\Phi_+\rangle = \sum_i |ii\rangle$ is the unnormalized maximally entangled state, $\mathcal{I}^{(R)}$ represents the identity channel acting on the reference system R , $\mathbb{I}$ is the identity matrix, and $\rho_{\mathcal{U}} = \mathcal{U} \otimes \mathcal{I}(\Phi_+)$ is the unnormalized Choi state of $\mathcal{U}$.

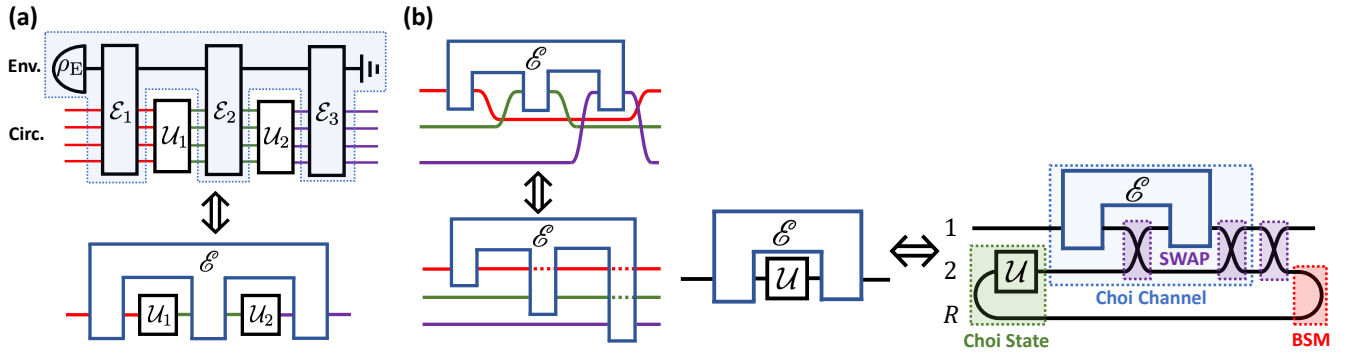


FIG. 1. **Left:** (a) Non-Markovian noise caused by interaction with the same environment system at different time steps, where wires in different colours represent systems from different time steps. (b) Shifting the input and output wires of the same time step to the same register outputs the Choi channel. A simplified graph representation is shown in the lower panel. The figure here only shows two circuit layers, which can easily be generalised into an arbitrary number of circuit layers. **Right:** The mapping between non-Markovian quantum dynamics and its Choi channel. BSM stands for Bell state measurement.

While our demonstration focuses on superchannels, representing the sequential composition of two successive quantum processes, our framework is broadly applicable and readily extends to address non-Markovian noise arising from the composition of arbitrary finite quantum processes.

Mitigating non-Markovian noise. If we know the exact form of the error channel $\mathcal{E}$, we can try to apply its inverse to mitigate the damage caused by the noise. However, such an inverse channel is often not completely positive and trace-preserving and thus cannot be implemented physically. Probabilistic error cancellation is a way to virtually implement the inverse map $\mathcal{E}^{-1}$ through linear combination of physically implementable operations $\{\mathcal{A}_i\}$ with post-processing [29, 30], as shown in Fig. 2Left(a).

Using the Choi channel representation of the non-Markovian noise $\mathcal{E}$, we can straightforwardly obtain its inverse map $\mathcal{C}_{\mathcal{E}}^{-1}$. It can be decomposed into basis operations generated using Z -basis measurement, S gate and Hadamard as outlined in [30], which means that these basis operators can be written as tensor products of local basis in each register, denoted using $\{\mathcal{A}_i\}$ and $\{\mathcal{B}_j\}$:

$$\mathcal{C}_{\mathcal{E}}^{-1} = \sum_{ij} \alpha_{ij} \mathcal{A}_i \otimes \mathcal{B}_j. \quad (3)$$

With such a tensor representation, we can directly translate this inverse Choi channel back into the circuit picture to implement the inverse superchannel:

$$\mathcal{E}^{-1} \circ \mathcal{E}[\mathcal{U}] = \sum_{i,j} \alpha_{ij} \mathcal{B}_j \circ \mathcal{E}[\mathcal{U} \circ \mathcal{A}_i] \quad (4)$$

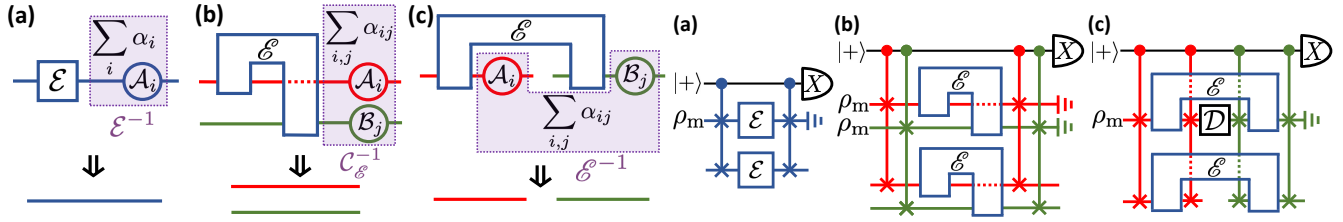


FIG. 2. **Left:** Probabilistic error cancellation for (a) noise channel; (b) Choi channel; (c) non-Markovian noise. **Right:** Virtual purification in the level of (a) noise channel; (b) noise Choi channel; (c) non-Markovian noise. Here, ρ_m represents the maximally mixed state and $\mathcal{D}$ is the completely depolarising channel.

which is shown in Fig. 2Left(b) and (c).

Given the challenges associated with characterising non-Markovian noise, one may want to turn to QEM techniques that do not require detailed knowledge of the noise model and the target problem. Virtual channel purification [33] is one such technique, relying only on the assumption that the dominant component of the target noise channel is the identity channel. The circuit for implementing virtual channel purification is shown in Fig. 2Right(a), where we conjugate two copies of the target noise channel with two CSWAP gates. One copy of the noise channel acts on a maximally mixed input state, which can be prepared by random initialisation on the ancillary register. The output post-processed based on the control qubit Pauli- X measurement result can suppress the noise rate from p to p^2 on the main register. By extending virtual channel purification to using M copies of noisy channels, the noise rate can be suppressed from p to p^M , which is exponential with M .

We can directly apply virtual channel purification to the Choi channel of the non-Markovian noise as shown in Fig. 2Right(b) with each blue line expanded into two registers represented by the green and red lines and initialized as a larger maximally mixed state. When translating back into the circuit picture, we have CSWAPs conjugating each error layer of the non-Markovian noise, with a single control qubit connecting all four CSWAP. Part of the maximally mixed state in the Choi channel picture is replaced by a completely depolarising channel in the circuit picture (implemented by applying random Pauli gates). The post-processing performed in this case is exactly the same as Markovian noise. Let us use Pauli-diagonal non-Markovian noise (which can be obtained using Pauli twirling as discussed in the arXiv version) as an example to illustrate the effect of the virtual channel purification.

Theorem 1. *Given a Pauli-diagonal superchannel $\mathcal{E}[\mathcal{U}] = \sum_{i,j} p_{ij} \mathcal{G}_i \circ \mathcal{U} \circ \mathcal{G}_j$, performing the circuit shown in Fig. 2Right(c) and taking the difference between the $|+\rangle$ and $|-\rangle$ measurement results of the control qubit, one virtually obtains the purified superchannel of*

$$\mathcal{E}_{\text{vp}}[\mathcal{U}] = \sum_{i,j} \frac{p_{ij}^2}{P_2} \mathcal{G}_i \circ \mathcal{U} \circ \mathcal{G}_j, \quad (5)$$

on the main register, where $P_2 = \sum_{i,j} p_{ij}^2$. If post-selecting on the $|+\rangle$ measurement results of the control qubit, one can physically obtain the purified channel

$$\mathcal{E}_{\text{p}}[\mathcal{U}] = \sum_{i,j} \frac{p_{ij} + p_{ij}^2}{1 + P_2} \mathcal{G}_i \circ \mathcal{U} \circ \mathcal{G}_j. \quad (6)$$

Without loss of generality, let us suppose $\mathcal{G}_0 \circ (\cdot) \circ \mathcal{G}_0$ is the noiseless component that we wish to boost. According to this theorem, if the noiseless component is the principal component, i.e., $p_{00} \geq p_{ij}$ for $i, j \neq 0$, then it will be amplified after both purification processes. Also note even though we have assumed the two copies of the non-Markovian noise to be the same in the above analysis, our protocol for non-Markovian noise purification will still work as long as the principal components in these two non-Markovian noise processes are similar, as also proven for the channel case in Ref. [33].

-
- [1] L. Viola, E. Knill, and S. Lloyd, Dynamical Decoupling of Open Quantum Systems, *Phys. Rev. Lett.* **82**, 2417 (1999).
 - [2] B. M. Terhal, Quantum error correction for quantum memories, *Reviews of Modern Physics* **87**, 307 (2015).
 - [3] S. J. Devitt, W. J. Munro, and K. Nemoto, Quantum error correction for beginners, *Reports on Progress in Physics* **76**, 076001 (2013).
 - [4] Z. Cai, R. Babbush, S. C. Benjamin, S. Endo, W. J. Huggins, Y. Li, J. R. McClean, and T. E. O'Brien, Quantum error mitigation, *Rev. Mod. Phys.* **95**, 045005 (2023).
 - [5] Ángel Rivas, S. F. Huelga, and M. B. Plenio, Quantum non-markovianity: characterization, quantification and detection, *Reports on Progress in Physics* **77**, 094001 (2014).
 - [6] U. Shrikant and P. Mandayam, Quantum non-markovianity: Overview and recent developments, *Frontiers in Quantum Science and Technology* **2**, 10.3389/frqst.2023.1134583 (2023).
 - [7] P. Taranto, M. T. Quintino, M. Murao, and S. Milz, Characterising the Hierarchy of Multi-time Quantum Processes with Classical Memory, *Quantum* **8**, 1328 (2024).
 - [8] B. M. Terhal and G. Burkard, Fault-tolerant quantum computation for local non-markovian noise, *Phys. Rev. A* **71**, 012336 (2005).
 - [9] M. J. Biercuk, H. Uys, A. P. VanDevender, N. Shiga, W. M. Itano, and J. J. Bollinger, Optimized dynamical decoupling in a model quantum memory, *Nature* **458**, 996 (2009).
 - [10] G. D. Berk, S. Milz, F. A. Pollock, and K. Modi, Extracting quantum dynamical resources: consumption of non-markovianity for noise reduction, *npj Quantum Information* **9**, 104 (2023).
 - [11] J. Preskill, Sufficient condition on noise correlations for scalable quantum computing, *Quantum Info. Comput.* **13**, 181 (2013).
 - [12] H. Bombín, Resilience to time-correlated noise in quantum computation, *Phys. Rev. X* **6**, 041034 (2016).
 - [13] A. Tanggara, M. Gu, and K. Bharti, Strategic code: A unified spatio-temporal framework for quantum error-correction, *arXiv preprint arXiv:2405.17567* (2024).
 - [14] H. Hakoshima, Y. Matsuzaki, and S. Endo, Relationship between costs for quantum error mitigation and non-Markovian measures, *Phys. Rev. A* **103**, 012611 (2021).
 - [15] G. Gutoski and J. Watrous, Toward a general theory of quantum games, in *Proceedings of the Thirty-Ninth Annual ACM Symposium on Theory of Computing*, STOC '07 (Association for Computing Machinery, New York, NY, USA, 2007) p. 565–574.
 - [16] G. Chiribella, G. M. D'Ariano, and P. Perinotti, Quantum circuit architecture, *Phys. Rev. Lett.* **101**, 060401 (2008).
 - [17] G. Chiribella, G. M. D'Ariano, and P. Perinotti, Theoretical framework for quantum networks, *Phys. Rev. A* **80**, 022339 (2009).
 - [18] F. A. Pollock, C. Rodríguez-Rosario, T. Frauenheim, M. Paternostro, and K. Modi, Non-markovian quantum processes: Complete framework and efficient characterization, *Phys. Rev. A* **97**, 012127 (2018).
 - [19] S. Milz and K. Modi, Quantum stochastic processes and quantum non-markovian phenomena, *PRX Quantum* **2**, 030201 (2021).
 - [20] J. Xing, T. Feng, Z. Fan, H. Ma, K. Bharti, D. E. Koh, and Y. Xiao, Fundamental limitations on communication over a quantum network, *arXiv preprint arXiv:2306.04983* (2023).
 - [21] Y. Xiao, Y. Yang, X. Wang, Q. Liu, and M. Gu, Quantum uncertainty principles for measurements with interventions, *Phys. Rev. Lett.* **130**, 240201 (2023).
 - [22] M. A. Nielsen and I. L. Chuang, *Quantum Computation and Quantum Information: 10th Anniversary Edition* (Cambridge University Press, 2010).
 - [23] M. M. Wilde, *Quantum Information Theory* (Cambridge University Press, 2013).
 - [24] J. Watrous, *The Theory of Quantum Information* (Cambridge University Press, 2018).
 - [25] Y. Yang, Memory effects in quantum metrology, *Phys. Rev. Lett.* **123**, 110501 (2019).
 - [26] Y. Xiao, K. Sengupta, S. Yang, and G. Gour, Uncertainty principle of quantum processes, *Phys. Rev. Res.* **3**, 023077 (2021).
 - [27] A. Altherr and Y. Yang, Quantum metrology for non-markovian processes, *Phys. Rev. Lett.* **127**, 060501 (2021).
 - [28] A. Raza, M. C. Caro, J. Eisert, and S. Khatri, Online learning of quantum processes, *arXiv preprint arXiv:2406.04250* (2024).
 - [29] K. Temme, S. Bravyi, and J. M. Gambetta, Error Mitigation for Short-Depth Quantum Circuits, *Phys. Rev. Lett.* **119**, 180509 (2017).
 - [30] S. Endo, S. C. Benjamin, and Y. Li, Practical Quantum Error Mitigation for Near-Future Applications, *Phys. Rev. X* **8**, 031027 (2018).
 - [31] W. J. Huggins, S. McArdle, T. E. O'Brien, J. Lee, N. C. Rubin, S. Boixo, K. B. Whaley, R. Babbush, and J. R. McClean,

- Virtual Distillation for Quantum Error Mitigation, *Phys. Rev. X* **11**, 041036 (2021).
- [32] B. Koczor, Exponential Error Suppression for Near-Term Quantum Devices, *Phys. Rev. X* **11**, 031057 (2021).
 - [33] Z. Liu, X. Zhang, Y.-Y. Fei, and Z. Cai, Virtual channel purification, *arXiv preprint arXiv:2402.07866* (2024).
 - [34] P. Pechukas, Reduced dynamics need not be completely positive, *Phys. Rev. Lett.* **73**, 1060 (1994).
 - [35] R. Alicki, Comment on “reduced dynamics need not be completely positive”, *Phys. Rev. Lett.* **75**, 3020 (1995).
 - [36] P. Pechukas, Pechukas replies:, *Phys. Rev. Lett.* **75**, 3021 (1995).
 - [37] G. Chiribella, G. M. D’Ariano, and P. Perinotti, Transforming quantum operations: Quantum supermaps, *Europhysics Letters* **83**, 30004 (2008).