

Tangle-Nonlocality Signatures in Structured Ansatz Ensembles

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Abstract

Nonlocality certifies entanglement and generalizes to multipartite scenarios. While the two-party case is well characterized, quantifying entanglement beyond two parties is considerably more involved. In the tripartite case, one can visualize the relationship between tripartite entanglement described by the three-tangle [1] and multipartite Bell nonlocality. In this work we find that expressivity-restricted ansatz ensembles exhibit distinct, structured coverage patterns within the attainable region of three-tangle and maximized violation degree, with some *ansätze* achieving dense coverage of specific subregions even at relatively low circuit depth – a result with direct implications for the distribution of non-classical resources. To study this, we generate pure quantum states from ensembles defined by parametrized ansatz circuits and examine how they populate this region, previously shown to form a wedge-shaped region [2–4]. This may allow for better selection of ansatz circuits based on non-classical correlations, e.g., for benchmarking [5, 6].

Bell inequalities like Svetlichny’s and Mermin’s allow for witnessing (multipartite) entanglement, and their violation implies nonlocality, with the distinction that Svetlichny’s inequality is violated only by genuinely multipartite nonlocal states, as opposed to Mermin’s [7]. Focusing on the tripartite case, Mermin’s inequality [8] is given as:

$$M_3 = \frac{1}{2} \left(A_1^{(1)} A_0^{(2)} A_0^{(3)} + A_0^{(1)} A_1^{(2)} A_0^{(3)} + A_0^{(1)} A_0^{(2)} A_1^{(3)} - A_1^{(1)} A_1^{(2)} A_1^{(3)} \right). \quad (1)$$

Here, $A_s^{(k)} = \hat{a}_s^{(k)} \cdot \vec{\sigma}^{(k)}$ denotes the *dichotomic* observable of party $k \in \{1, 2, 3\}$ for measurement setting $s \in \{0, 1\}$, defined by a unit vector $\hat{a}_s^{(k)} \in S^2$. For the generalization to n -qubits we refer to [8–11]. Svetlichny’s inequality [12, 13] is given in the tripartite case as:

$$\begin{aligned} S_3 = & A_0^{(1)} A_0^{(2)} A_0^{(3)} + A_0^{(1)} A_0^{(2)} A_1^{(3)} + A_0^{(1)} A_1^{(2)} A_0^{(3)} + A_1^{(1)} A_0^{(2)} A_0^{(3)} \\ & - A_1^{(1)} A_1^{(2)} A_0^{(3)} - A_1^{(1)} A_0^{(2)} A_1^{(3)} - A_0^{(1)} A_1^{(2)} A_1^{(3)} - A_1^{(1)} A_1^{(2)} A_1^{(3)}. \end{aligned} \quad (2)$$

Considering the tripartite case, for Mermin any local hidden variable model obeys $|\langle M_3 \rangle| \leq 1$ and for Svetlichny, the biseparable (hybrid local-nonlocal) bound is given as $|\langle S_3 \rangle| \leq 4$, while the upper bounds achievable in quantum mechanics are stated as $|\langle M_3 \rangle| \leq 2$ and for Svetlichny as $|\langle S_3 \rangle| \leq 4\sqrt{2}$.

Here, we particularly focus our investigation on a specific family of circuits and apply multiple restrictions. The ansatz circuit parameters are initialized at random in order to compare the region that is reachable per (restricted) ansatz ensemble. With the indication of rotation axis

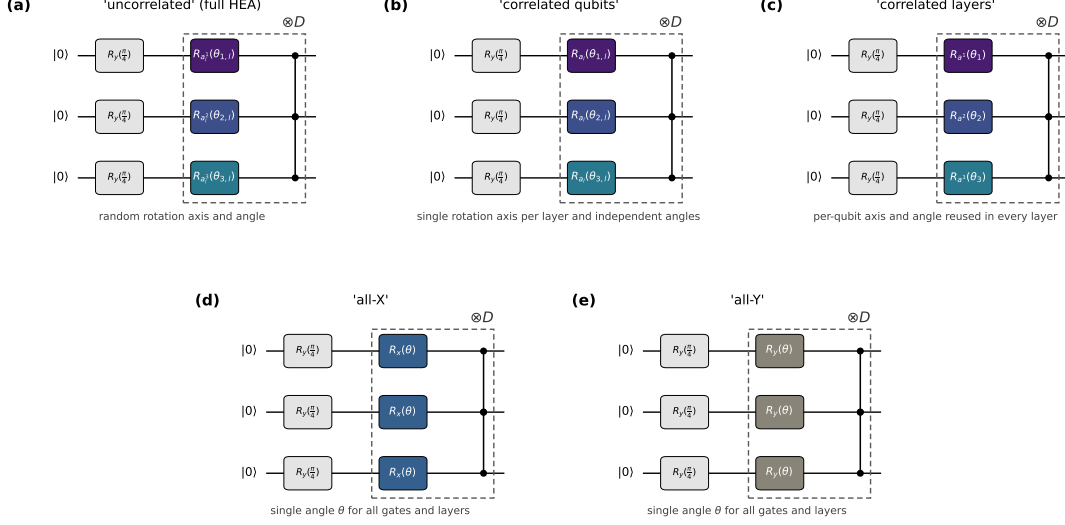


Figure 1: Restricting the HEA ansatz circuit to reduce the expressivity of the ensembles used in this analysis. The depth D defines the number of repetitions of the layer of single-qubit rotations and CZ gates, arranged in a ladder structure. Colors indicate whether the angles/rotation axes of each gate differ or not. The expressivity-restriction approach follows [14].

$\sigma_{a_l^i} \in \{X, Y, Z\}$, the parametrized unitary defining the ansatz for D layers and n qubits is defined as [14]:

$$U_{\text{HEA}}(\mathbf{a}_l, \theta_l, D) := \prod_{l=1}^D \left(\left(\prod_{i=1}^{n-1} \text{CZ}_{i,i+1} \right) \left(\bigotimes_{i=1}^n e^{-i\theta_l^i \sigma_{a_l^i}/2} \right) \right), \quad (3)$$

and is applied to the initial state $(R_y(\pi/4)|0\rangle)^{\otimes n}$. This generic ansatz family is restricted in multiple ways as visualized in Figure 1 in order to reduce its expressivity, and the rotation angles are sampled uniformly i.i.d., i.e., $\theta \sim \mathcal{U}([0, 2\pi))$ while the rotation axes are drawn uniformly from $\{X, Y, Z\}$ if they are not set constantly. For a given state $|\psi\rangle$, the maximal violation degree is computed by optimizing over all possible measurement directions as in [15] with multiple restarts, i.e.,

$$|\langle \mathcal{B}_3 \rangle|_{\max} = \max_{\{\hat{a}_s^{(k)} \in S^2\}} |\langle \psi | \mathcal{B}_3 | \psi \rangle|, \quad \mathcal{B}_3 \in \{M_3, S_3\}. \quad (4)$$

The three-tangle [1], quantifying tripartite entanglement, is defined as $\tau_3 = \tau_{A|BC} - \tau_{AB} - \tau_{AC}$, with $\tau_{A|BC} = 2(1 - \text{Tr}(\rho_A^2))$ and $\tau_{AB} = C^2(\rho_{AB})$, $\tau_{AC} = C^2(\rho_{AC})$ the squared concurrences [16]. The three-tangle $\tau_3 \in [0, 1]$ is maximal for GHZ states.

In the depicted tripartite case as in Eqs. (1) and (2), one can investigate how the ansatz ensemble explores the tangle-nonlocality relationship for Svetlichny's and Mermin's inequality [2, 3]. The outer bounds, i.e., the general wedge structure, were found to be dependent on the specific witness that was used, e.g., Svetlichny or Mermin [2–4]. So far, the focus has been on the boundaries of certain families of states as shown in Figure 2, but recent work has started to also investigate the interior of the tangle-nonlocality wedge for quantum random circuits [15]. Here, we aim to investigate the interior region in a structured way by considering the different ansatz ensembles as in Figure 1 that exhibit different levels of expressivity as defined in [17] in order to examine their resource distribution. This lets us investigate how (multipartite) entanglement and nonlocality are distributed depending on the ensemble under consideration. It is also of interest which ensembles

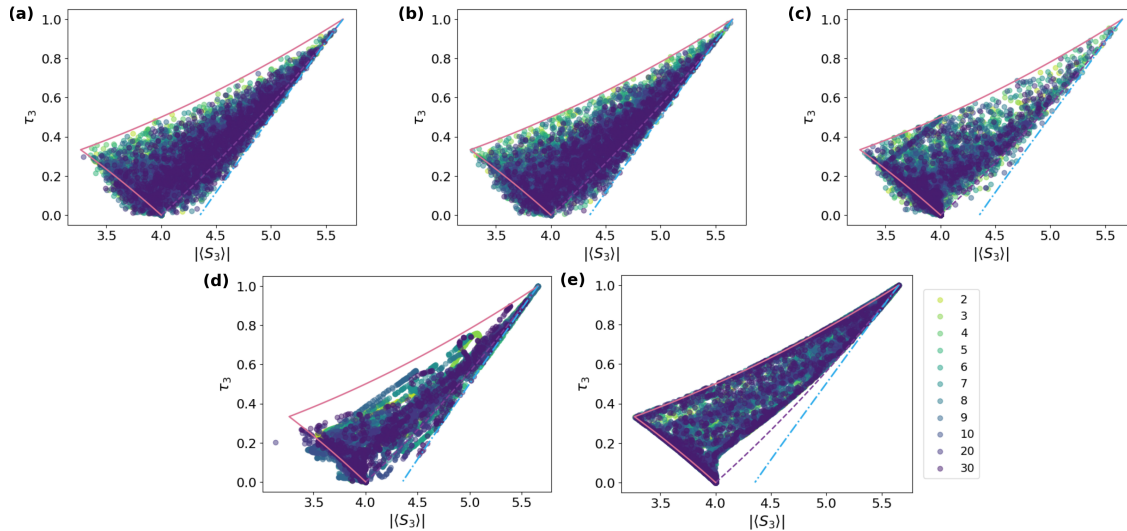


Figure 2: Maximal violation degree versus three-tangle τ_3 for the five ensembles of Fig. 1, where the labels (a) to (e) match the respective circuit structures, with $n = 3$ and 10^3 states per depth. Colors indicate the depth $2 \leq D \leq 30$ as depicted in the legend. The magenta curve shows the generalized-GHZ boundary and the dashed purple one the maximal-slice reference family [2, 3], while the dash-dotted blue curve refers to the family of tetrahedral states [4]. The bound $|\langle S_3 \rangle| = 4$ marks the onset of genuine multipartite nonlocality, while the quantum maximum is $4\sqrt{2}$.

reach the high- and intermediate-resource regions at lower circuit depth and cover the (full) wedge.

As visible in Figure 2, specifically, the one-parameter family per depth (e) containing only real states densely covers the region adjacent to the generalized GHZ boundary, while including the maximally entangled states and leaving the low-tangle intermediate violation degree corner empty. In contrast, the ensemble which is also a one-parameter family at a fixed depth D in (d) does not cover the high three-tangle region. This structured exclusion is absent from the more expressive ensembles (a)-(c), while the lower three-tangle regions are more populated at higher depth D . The same qualitative taxonomy appears also in the Mermin case, suggesting a witness-independence of the observed patterns¹.

In conclusion, in this work we have investigated how the accessible tangle-nonlocality region is covered for different state ensembles with maximized Bell inequality violation degree, focusing on Svetlichny and Mermin. This was motivated by the search for low-depth ansatz circuits that allow for improved quantum hardware benchmarking protocols based on nonlocality [5, 6, 15]. Furthermore, ensemble-specific fingerprints are extracted due to the different patterns that are visible. Of course, the depicted analysis is with respect to three parties, and it is known that an analysis becomes increasingly difficult with a higher number of parties and in the mixed-state case, even deciding whether a general mixed state is entangled or separable is NP-hard [19]. Recent learning-based approaches have demonstrated that entanglement signatures based on the partial transpose can be inferred from few measurements even in higher-dimensional systems [20], offering a possible extension.

¹The full extended analysis can be found in [18].

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