

Weighted Quantum Signal Processing for Kolmogorov–Arnold Networks: Efficient Activation Function Realization and Quantum Resource Bounds

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Abstract. Quantum Signal Processing (QSP) is a powerful approach for generating and approximating univariate polynomials but is limited by increasing circuit depth and parity restrictions. We introduce Weighted Quantum Signal Processing (WQSP), a generalization of QSP that incorporates fixed weights into signal operators, with standard QSP recovered as the unit-weight case. Classical weight encoding enables efficient realization of high-degree polynomials with linear to exponentially reduced circuit resources. We apply this approach to realize activation functions in Kolmogorov–Arnold Networks (KANs) for multivariate function approximation. WQSP-based KANs outperform Fourier KANs, QSP-KANs, and other KAN variants while providing a compact, and resource-efficient architecture for polynomial and function realization suitable for NISQ-compatible neural architectures.

Key words. Weighted Quantum Signal Processing, Chebyshev Polynomials, Quantum circuit, Kolmogorov–Arnold Networks.

1 Introduction

Artificial neural networks, particularly multilayer perceptrons (MLPs), are powerful function approximators owing to the universal approximation theorem [6]. However, their effectiveness is often limited by the curse of dimensionality, motivating alternative architectures that exploit structure in high-dimensional data. In this context, Kolmogorov–Arnold Networks (KANs) [10], inspired by the Kolmogorov–Arnold Representation Theorem (KART) [12, 1], have emerged as a promising alternative. KANs offer improved interpretability, expressivity, and data efficiency [10, 15, 13, 8, 4, 3, 16, 2] by replacing fixed activation functions with learnable univariate functions defined on network edges. Nevertheless, existing KAN implementations typically parameterize these functions using classical bases such as B-splines [10, 11], which can suffer from redundancy, initialization sensitivity, and rapidly growing parameter counts when approximating complex nonlinear functions [14].

Meanwhile, Quantum Signal Processing (QSP) provides a highly structured approach for implementing polynomial transformations through compact single-qubit circuits with minimal ancilla resources [5, 9]. QSP achieves expressive function approximation with favorable numerical stability, but standard formulations exhibit two key limitations, namely, the circuit depth and number of parameters scale with polynomial degree, and the realizable polynomial families are constrained by

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parity, requiring separate constructions for even and odd polynomials which further increases the parameter budget for constructing a generic polynomial.

To address these challenges, we introduce *Weighted Quantum Signal Processing* (WQSP), a natural extension of QSP that incorporates fixed integer weights into the signal operators. This formulation generalizes standard QSP, which is recovered as the special case of WQSP with all weights set to unity. These weights enable compressed realizations of high-degree polynomials with substantially reduced parameter counts and circuit depths, as pruned variants of QSP circuit. Motivated by these properties, we employ WQSP circuits as the learnable activation-function generators within KANs. We combine the compositional structure and interpretability of KANs with the expressive power of QSP approximation by representing each network-edge activation as a WQSP-generated polynomial. We derive approximation and resource bounds for the resulting architecture and show that WQSP-assisted KANs provide a compact, expressive, and numerically stable approach for approximating multivariate continuous functions, achieving significant reductions in parameter complexity compared with QSP-based KANs while maintaining strong approximation performance.

2 Weighted Quantum Signal Processing

In this section, we provide a mathematical overview of WQSP. Let us consider $\mathbf{w} = (w_1, \dots, w_k)^T \in \mathbb{R}^k$ and $x = \cos \theta$. Then we define WQSP as $U_{\Phi_{\mathbf{w}}}^{\text{WQSP}}(x) = \left(\prod_{j=1}^k R_z(\phi_j) \exp(iw_j\theta\sigma_x) \right) R_z(\phi_0)$. Clearly, when $w_j = 1 \forall j \in \{1, \dots, k\}$, the expression reduces to QSP. Restricting $\mathbf{w}$ to $\mathbb{N}^k$ admits an interpretation of WQSP as a structured and pruned variant of QSP, where setting selected phase angles $\phi_j = 0$ merges adjacent signal operators into powers $U(x)^{w_j}$ and effectively removes intermediate R_z rotations, producing parameter-efficient WQSP circuits. Similar to QSP, WQSP also admits the following $U_{\Phi_{\mathbf{w}}}^{\text{WQSP}}(x) = \begin{bmatrix} P(x) & iQ(x)\sqrt{1-x^2} \\ iQ(x)^*\sqrt{1-x^2} & P(x)^* \end{bmatrix}$ where $P(x)$ is a linear combination of Chebyshev polynomials for appropriate weights. Hence, with suitable weight vectors, a given polynomial can be realized while bypassing the parameter-heavy constraints of standard QSP. Using a learning-based architecture, we show that approximating $P(x)$ with an WQSP circuit reveals significant parameter redundancy in QSP and highlights the advantage of exploiting polynomial structure.

3 WQSP for Kolmogorov-Arnold Networks

Kolmogorov–Arnold Networks (KANs) are neural network architectures inspired by the Kolmogorov–Arnold Representation Theorem (KART) [10]. Unlike conventional networks, KANs place learnable univariate functions on edges and perform summation at nodes. KART, originating from Hilbert’s 13th problem, states that any continuous function on a compact subset of $\mathbb{R}^n$ can be represented as $f(x_1, \dots, x_n) = \sum_{q=0}^{2n} \phi_q \left(\sum_{p=1}^n \psi_{pq}(x_p) \right)$, where ϕ_q and ψ_{pq} are univariate continuous functions, with ψ_{pq} independent of f . This representation motivates the KAN architecture, providing enhanced expressivity and interpretability for scientific and mathematical applications [10, 15, 13]. Quantum KANs were first introduced in [7] via block-encoding. Here, we incorporate different WQSP regimes to realize nonlinear activation functions in KANs as depicted in Figure 1.

We approximate a target function (Figure 3(a)) using WQSP-assisted KANs with degree-10 polynomial activations, realized by WQSP circuits whose signal operator weights correspond to all 42 different integer partitions of 10. Additionally, Figure 2 depicts the approximation errors and parameter cost of all the QKANs constructed with different WQSP weight choices (partitions of 10), highlighting the corresponding partition index (Figure 2(b)).

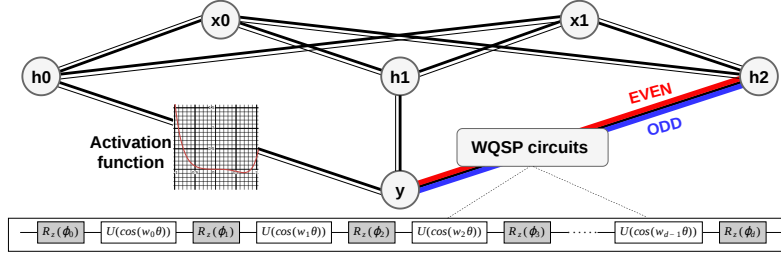


Figure 1: The proposed WQSP-based QKAN architecture. Each activation edge carries two WQSP circuits of depths L and $L - 1$ approximating the even and odd parity components of the activation function respectively, whose outputs are summed to produce an arbitrary univariate activation.

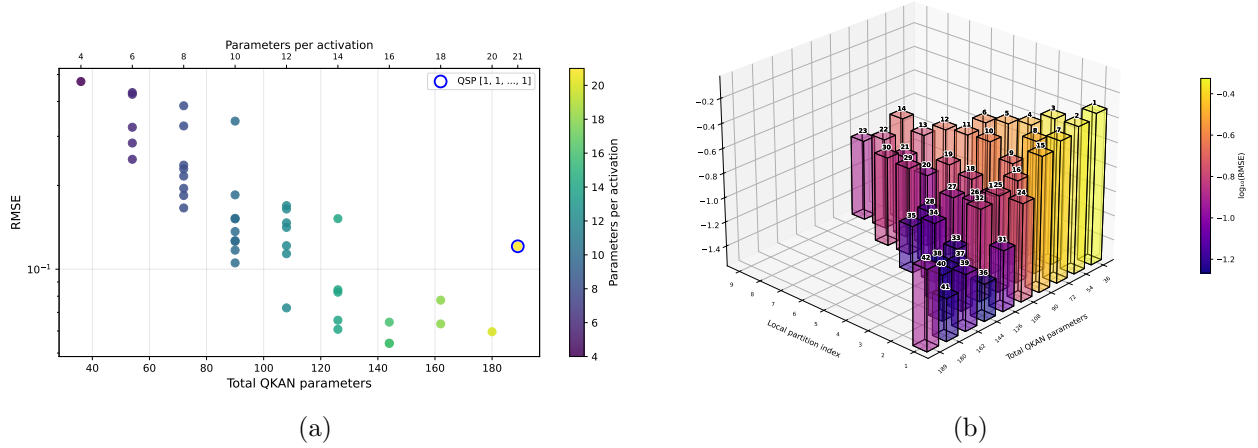


Figure 2: (a) RMSE vs. parameter count for a $[2, 3, 1]$ QKAN with degree-10 activations using WQSP across all integer partitions of 10. Several partitions match or outperform QSP with fewer parameters. (b) 3D bar plot highlighting each individual partition of 10 by their corresponding index.

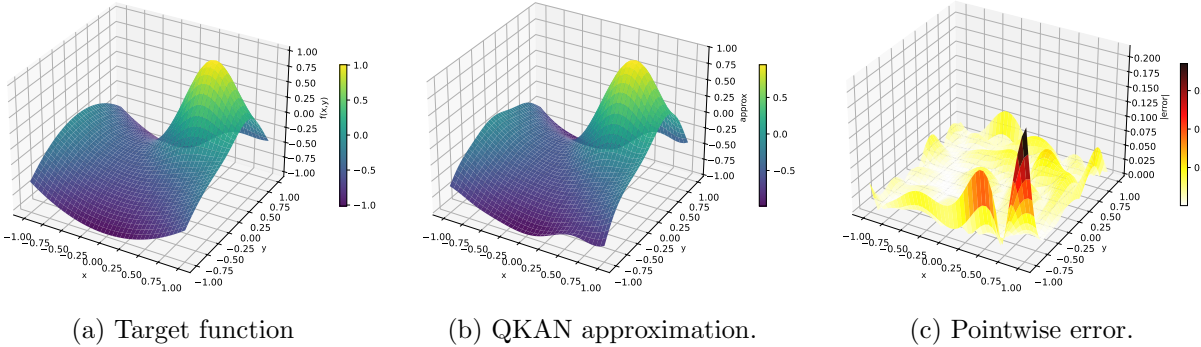


Figure 3: One instance of multivariate function approximation by a WQSP-QKAN.

Conclusion. We introduced WQSP-based QKANs and identified parameter-efficient WQSP configurations that outperform QSP-QKAN for multivariate function approximation, demonstrating that exploiting weight structure in signal operators can substantially reduce circuit depth and parameter overhead. This points to a broader design space for resource-efficient QKAN architectures, which we plan to investigate further, extending to higher-dimensional inputs, richer function classes, and diverse machine learning tasks in future work.

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