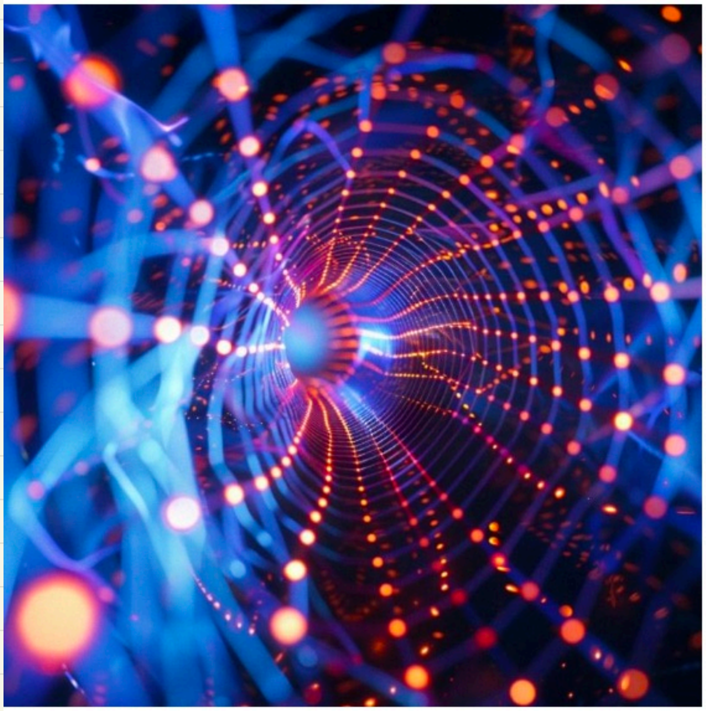
South African Quantum Technology Initiative

NITheCS
National Institute for Theoretical and Computational Sciences

International Year of Quantum Science and Technology

-- 33rd Chris Engelbrecht Summer School 2025 --



Theoretical Foundations of Quantum Science and Quantum Technologies

STIAS, Stellenbosch, South Africa

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Exercises for lectures on

Open quantum systems

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1 Let $\phi(t) = c_0 \psi_0 + c_1(t)\psi_1 + \sum_k d_k(t)\chi_k$

be the state (in the interaction picture) of the atom-cavity complex in the Jaynes–Cummings model. Show that

$$\mathrm{tr}_B\left(|\phi(t)\rangle\langle\phi(t)| \right)$$

$$= \left(|c_0|^2 + \sum_k |d_k(t)|^2 \right) |0\rangle\langle 0| + |c_1(t)|^2 |1\rangle\langle 1|$$

$$+ c_0 \overline{c_1(t)} |0\rangle\langle 1| + \overline{c_0}\, c_1(t) |1\rangle\langle 0|.$$

Note: $1 = \|\phi(t)\|^2 = |c_0|^2 + |c_1(t)|^2 + \sum_k |d_k(t)|^2$

So $|c_0|^2 + \sum_k |d_k(t)|^2 = 1 - |c_1(t)|^2$

② For the Jaynes–Cummings model the reduced atom density matrix (interaction picture) satisfies:

$$\partial_t \rho_{S,I}(t) = \begin{pmatrix} -\gamma(t)\, |c_1(t)|^2 & -\frac{1}{2}\bar{c}_0 \left(\gamma(t) + i S(t)\right) c_1(t) \\ \frac{1}{2} c_0 \left(\gamma(t) - i S(t)\right) \overline{c_1(t)} & \gamma(t)\, |c_1(t)|^2 \end{pmatrix}$$

(Matrix written in orthonormal basis $\{|1\rangle, |0\rangle\}$.)

Show that

$$\partial_t \rho_{S,I}(t) =$$

$$-\frac{i}{2} S(t) \left[\sigma_+ \sigma_-, \rho_{S,I}(t)\right]$$

$$+\gamma(t) \left\{ \sigma_- \rho_{S,I}(t) \sigma_+ - \frac{1}{2} \sigma_+ \sigma_- \rho_{S,I}(t) - \frac{1}{2} \rho_{S,I}(t) \sigma_+ \sigma_- \right\}$$